CHAPTER -1

MAGNETIC CIRCUIT & COUPLE CIRCUIT

Magneti circuit:-

Introduction:-

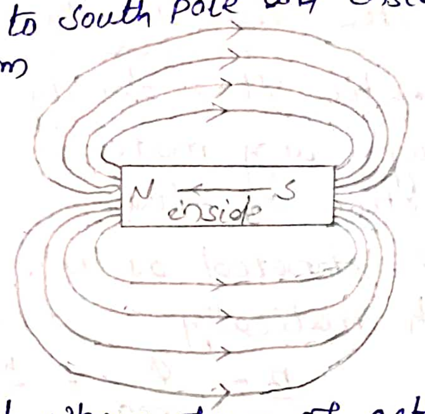
→ Magnets are found in the natural state in the form of material called magnetic.

→ Now a days natural magnets have no practical value. Because their magnetism is not strong enough to be utilise in the modern device.

→ Magnetism is the force exerted (present/show) by magnets when they attract or replay each other.

→ A magnet have two pole i.e, North pole and South pole.

→ By conversion the magnetic Lines of force flow on the out side any magnet form north pole to south pole and inside of a magnet the field lines flow form South to north.



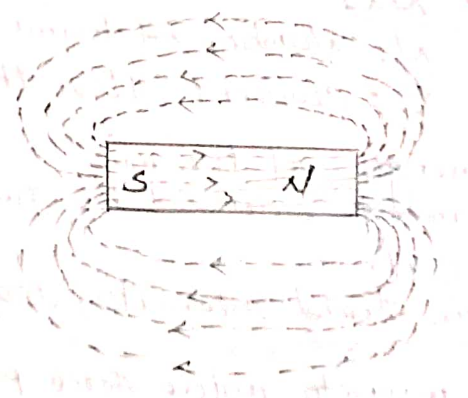
→ Like pole replaces each other & unlike pole attract each othere.

→ Magneti field lines exist (show) around a magnet.

→ The region (area) near the magnet where force of action of magnetic pole is called magnetic pole.

→ The magnetic field is strongest near the pole and goes on decreasing in strength as we move away form the magnet.

→ magnetic field around a magnet is represented by imaginary Lines is called magnetic lines of force or magnetic field Lines & they never crush each other.



Magnetic CKT:-

The representation of the system containing magnetic field, as an electrical network is called as Magnetic circuit.

Magnetic Flux:-

The amount of magnetic field produced by a magnetic source is called as magnetic flux.

Quantitative:-

It is denoted by ϕ

$\phi \propto$ field lines

SI unit of flux is Weber (wb)

1 weber = 10^8 lines

→ The close path followed by magnetic flux is called magnetic circuit.

Magnetic flux density:-

→ Magnetic flux density is the flux passing for unit area through any material through a right angle to a direction of flux.

→ It is denoted as 'B'.

→ Math matically

$$B = \frac{\phi}{area} = \frac{Weber}{mt^2}$$

→ SI unit of magnetic flux density (B) = $\frac{Wb}{mt^2}$

Magneto motive force (mmf):-

→ It required to derived the magnetic flux in the magnetic circuit.

→ The SI unit of mmf is Ampher turn.

→ mmf = N×I

N = Number of turns

I = Current flow through the magnetic cht.

→ The strength of the mmf is equivalent to the product of the current arround the turns and the number of turns of the coil.

Magnetic field Intencity / magnetising force (H):-

→ It is magneto motive force per unit area.

→ It is denoted by 'H'.

→ In mathmatically $H = \frac{mmf}{l}$

$$= \frac{ni}{l} = \frac{\text{Ampher turn per meter}}{\text{meter}} = \frac{A.T}{m}$$

→ SI unit of magentic field Intencity is $\frac{A.T}{m}$

Note:-

$$\phi = \frac{mmf}{Reluctance} = \frac{AT}{(l/a\ \mu_0\mu_r)}$$

$$AT = \phi \times \frac{l}{a \cdot \mu_0\mu_r}$$

$$= \frac{\phi}{a} \times \frac{l}{\mu_0\mu_r}$$

$$= B \times \frac{l}{\mu_0\mu_r}$$

$$\therefore AT = \frac{B}{\mu_0\mu_r} \times l$$

$$\therefore AT = H \times l$$

Relation between 'B' & 'H':-

→ $B \propto H$

→ $B = \mu H$ (μ = constant)

→ $\mu = \frac{B}{H}$ $\therefore \mu = \mu_0\mu_r$

so $\boxed{B = \mu_0\mu_r H}$

Where, μ = absolute permitbility of the material

μ_r = Relative permiability of the material

μ_0 = absolute permiability of air & vacum

→ In air or vacum, the relative permiability is 1 so $\mu = \mu_0$

Permeability:-

→ Permeability of a materal means its conductivity for magnetic flux.

→ Greater the permiability of a material, the greater is it's conductivity for magnetic flux and vice-versa.

→ Air or vacum is the poorest conductor of magnetic flux.

→ It is denoted by 'μ'

→ The absolute permeability (μ_0) of air or vacum (basic or standrad medium) is $4\pi \times 10^{-7}$ H/m or (Henery/meter). But the absolute permeability of magnetic material is much greater than μ_0.

- The ratio of μ/μ_0 is called the relative permeability of the material and is denoted by μ_r.

$$\boxed{\mu_r = \mu/\mu_0}$$

$$\boxed{\mu = \mu_r \cdot \mu_0}$$

- μ_r for air or vacuum would be $\frac{\mu}{\mu_0} = 1$

Extra Note :-
- The value of μ_r for all non-magnetic material is also 1.
- The value μ_r for magnetic material is very high.

Ex:- Iron = 8000 (pure iron)

Reluctance :-

- The opposition offers to magnetic flux by magnetic circuit is called reluctance.
- The reluctance in a magnetic cht depend open its length, area of cross-section and permeability of the material that make up the magnetic cht.
- It's unit is AT/weber

$$S = \frac{mmf}{\phi} = \frac{NI}{\phi} = \frac{l}{a\mu} = \frac{l}{a\mu_0\mu_r}$$

$$S = \frac{l}{a\mu_0\mu_r}$$

Permeance :- (It is reciprocal of conductor)

- It is reciprocal of reluctance which can passes flux through the material.
- It unit is Wb/At

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{a\mu}{l} = \frac{a\mu_0\mu_r}{l}$$

- It denoted by 'P'

Difference betw^n electric field & magnetic field

Electric field	Magnetic Field
Flow of current	Flow of magnetic flux.
Emf is the cause of flow of current.	mmf is the cause of flow of flux.
opposition offer to the flow of current is called resistance	opposition offer to the flow of magnetic field is reluctance.
Conductance = $\frac{1}{\text{Resistance}}$	Permeance = $\frac{1}{\text{Reluctance}}$
$I = \frac{Emf}{\text{Resistance}} = \frac{Emf}{R}$	$\phi = \frac{mmf}{\text{reluctance}} = \frac{mmf}{S}$

Electric field	Magnetic Field
Current actually flow in an electric cht.	Flux doesn't actually flow in an magnetic cht.
Energy is needed as long as current flow	Energy is initially neede to creat the magnetic flux but not to maintain it.
Conductance is constant and independent of current strength at a particular temp.	permeability depends on the total flux for a particular temp.

Series & Parallel magnetic Circuit :-

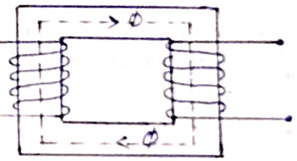
Series Combination :-

In Series Combination of a magnetic cht the magnetic flux through different parts of magnetic cht should be same.

$$S = S_1 + S_2 + S_3 \cdots S_n$$

$(S = \text{reluctance})$

Series Combination of magnetic Circuit :-

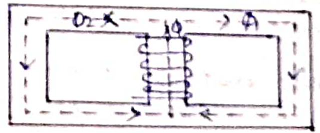


→ Core type

Parallel Combination :-

- In magnetic cht if flux gents devided into two or more than two parts than it is said to be parallel combination.

$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \cdots \frac{1}{S_n}$$



→ shell type

B.H Curve or Hysteresis Curve :-

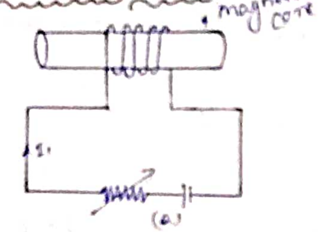
The salutation of magnetic circuit can be easily obtained by the use of B.H Curve.

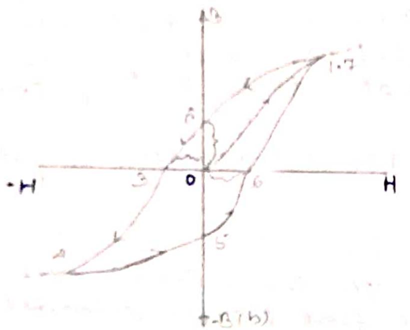
i) Corresponding to the flux density (B) in the material find the magnetising force "H" from B-H curve of the material.

ii) Compute the magnetic length 'l'.

iii) Then we can use for mmf = H×l but we don't use B-H curve to find mmf.

- **Historesis Loop:-**





i) From fig(a) when I = 0 i.e, $H = \frac{NI}{l} = 0$ and B = 0, this is represented by point 'O' in the graph.

ii) The current in the coil increase 'H' increases and so does 'B'. The 'BH' curve follows the path O-1 and point '1' is the beginning of magnetic salutation. Here all of the domains in the core material have become perfectly aligned.

→ If Now 'H' is reduced by decreasing current in the widge the curve follow the path '1-2'. Note that after magnetic saturatation has been reacted (point 1), then decreasing 'H' does not reduce 'B' along the same curve that it followed when it was increased.

→ At point '2', H = 0, but 'B' has finite value, this lagging of 'B' behind 'H' is called hysteresis. Actually the fact is the magnetic domain retain their alignment to same extent after the magnetising force is reduced or removed. So this flux density is said to be residual flux density (O-2), it remain in the core after the current is reduced to zero.

→ The degree to which magnetic material remains its magnetism after the magnetising force is reduced to zero is called Retentivity.

→ If direction of 'H' is reversed by reversing the current, and the B-H curve followed the path (2-3). Then a considerable amount of '-H' is required to retain the flux density to zero. The -ve value of 'H' required to reduced residual flux density to zero is called coercive force.

→ As 'H' increases further in the -ve direction, 'B' also reverse direction and increase negatively. Eventually 'H' becomes so large in the reverse direction that core again saturates at point '4'. We can say that point '4' is the mirror of point '1' and the magnitude of 'B' and 'H' are same but the reverse direction.

→ If 'H' is now reduced, 'B' decreases and follows the path (4-5). Note that point '5' is the residual flux density and same as to point 2.

→ If the direction of 'H' is now reverse so that is again positive. we see that 'B' remains -ve untill it is made sufficiently +ve to bring 'B' back zero (point 6). Further increase in 'H' causes 'B' to increase towards +ve saturation at point 1. The compute B-H curve 1-2-3-4-5-6-1 is called the hysteresis Loup.

CHAPTER - 2

COUPLED CIRCUIT

⇒ When the magnetic flux linking a conductor or coil changes, an emf is induced in that conductor.

⇒ If coil produced a flux 'ϕ' and the coil has 'n' no. of turns then the flux linking is

$$\psi = N\phi$$

$$\text{So, } e = N\frac{d\phi}{dt}$$

$$\text{or, } e = \frac{d}{dt}(N\phi) = \frac{d}{dt}(\psi)$$

$$e = -N\frac{d\phi}{dt} \quad \text{(According to lenz's law)}$$

Flux Linkage :-

⇒ Total flux passing through a coil of N turns.

⇒ The changes in flux linkage can be brought about in the following two ways.

1> Dynamically Induced emf
2> Statically Induced emf

1> **Dynamically Induced Emf :-**

The conductor is moved into the stationary field in such a way that the flux linking it, changes in magnitude. The emf induced in this way is called "dynamically induced emf."

→ In short form ⇒ Emf is produced in the conductor which in motion.

2> **Statically Induced Emf :-**

→ The conductor is stationary and the magnetic field is moving or changing. The emf induced in this way is called statically induced emf.

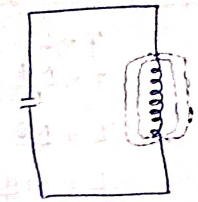
→ A statically induced emf can be further sub devided into

A> Self induced emf
B> Mutually induced emf

A> **Self Induced Emf :-**

⇒ The emf induced in a coil due to the change of his it's own flux linked with it is called self induced emf.

→ When current in a coil increases or decreases, there is a change in magnetic flux linking in the coil. Hence an emf induced in the coil. The process is called self-induced emf.



$$e = N\frac{d\phi}{dt}$$

→ According to lanz's law, the direction of this induced emf is such that it opposes the cause that has produced it. Now the cause of this induced emf is the cange in magnetic flux through the coil i.e change of current in the coil. Here the induced emf will opposes the change of current in the coil. This property of the coil is called its self-induced or inductance (L).

$$e = -N\frac{d\phi}{dt}$$

Self Inductance (L) :-

→ The property of a coil by virtue of which it opposes any change in the amount of current flowing through it, is called it's self inductance or inductance (L).

$$\Rightarrow e = -N\frac{d\phi}{dt}$$

$$\Rightarrow e = -\frac{d}{dt}(N\phi)$$

$$\Rightarrow N\phi \propto I$$

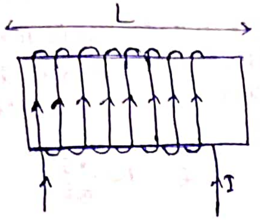
$$\Rightarrow \psi \propto I$$

$$\Rightarrow \psi = LI \quad (\therefore L = \text{proportionality constant})$$

→ it's unit is Henry.

$$\boxed{L = \frac{\psi}{I} = \frac{N\phi}{I}}$$

Inductance of Solenoid (or Coil):-



$$\psi = LI$$
$$I = \frac{N\phi}{L} = \frac{\psi}{L}$$
$$L = \frac{N\phi}{I}$$
$$L = N\frac{d\phi}{dI}$$

Actually, $\phi = \frac{mmf}{reluctance} = \frac{NI}{\frac{l}{a\mu_0\mu_r}}$

Area of cross Section.

Differentiating ϕ w.r.t. we get

$$= \frac{d\phi}{dI} = \frac{N a\mu_0\mu_r}{l}$$
$$= \frac{L}{N} = \frac{N a\mu_0\mu_r}{l}$$
$$= L = \frac{N^2 a\mu_0\mu_r}{l}$$
$$\Rightarrow L = \frac{N^2}{\frac{l}{a\mu_0\mu_r}} = \frac{N^2}{Reluctance(S)}$$

→ Induced is directly proportional to turns squared and Inversely proportional to the reluctance of the magnetic path.

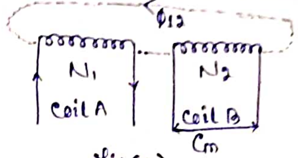
→ An Iron-Cored coil has more inductance then the equivalent air cored coil.

→ A coil is said to have large Self Inductance if it produces a large Induced emf.

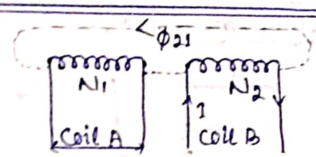
→ The value of "L" depends upon the dimensions of the coil, no. of turns and the relative permeability of the core material.

2) Mutually Induced emf =

→ The induced emf in a coil due to the changing current in the neighbouring coil is called mutually Induced emf (em).



fig(a)



fig(b)

→ Consider the above two coil's A&B placed near each other. Coil "B" is not electrically connected coil "A".

→ If current "I_1" flowing through coil "A", the magnetic field is set up and the part of (ϕ_{12}) this flux links with Coil "B". it current in coil "A" is changed, the mutual flux also changes and hence an emf is induced in Coil "B". The emf Induced in coil "B" is named as mutually Induced emf. and the process is known as mutual Induction.

→ According to Faraday's law of electromagnetic Induction

$$e_m = -N_2\frac{d\phi_{12}}{dt}$$

Similarly

$$e_m = \frac{-N_1\phi_{21}}{dt}$$

Mutual Inductance (M):-

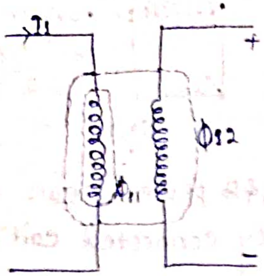
→ The property of two coils by virtual of which each opposes any change of current flowing in the other is called mutual inductance bet^n two coil.

→ This opposition occurs because a changing current in one coil produces mutually induced emf in the other coil which opposes the changing of current in the first coil.

$$N_1\phi_{21} \propto I_2$$
$$N_2\phi_{12} \propto I_1$$
$$e_{m1} \propto -\frac{dI_1}{dt}$$
$$e_{m2} \propto -\frac{dI_1}{dt}$$

$$\boxed{\begin{aligned} e_{m1} &= -m\frac{dI_1}{dt} \\ e_{m2} &= -m\frac{dI_2}{dt} \end{aligned}}$$

Mutual Inductance (M) :-



$\boxed{\phi_1 = \phi_{11} + \phi_{12}}$

$\phi_{12} \rightarrow$ Mutual flux from coil 1 to coil 2.

$emf_{coil_2} = -m \frac{dI_1}{dt}$

$\rightarrow emf_{coil_2} = \frac{-N_2 \, d\phi_{12}}{dt}$

$\rightarrow -m \frac{dI_1}{dt} = -N_2 \frac{d\phi_{12}}{dt}$

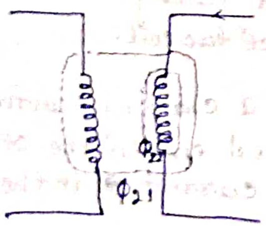
$\rightarrow \cancel{\frac{d}{dt}} (MI_1) = N_2 \, \phi_{12} \cancel{\frac{d}{dt}}$

$\rightarrow MI_1 = N_2 \phi_{12}$

$\rightarrow \boxed{M = \frac{N_2 \phi_{12}}{I_1}}$

→ Mutual Inductance depends on mutual flux.

Similarly from current I_2



$\boxed{M = \frac{N_1 \phi_{21}}{I_2}}$ $\quad \frac{web}{Amp} = $ Henery

Coefficient of coupling (K) :-

→ It represent the flux linking capacity of coil.

or

→ The fraction of total flux links the other coil, is represent by co-efficient of coupling.

$K = \frac{\phi_{12}}{\phi_1} = \frac{\text{Mutual flux}}{\text{total flux}}$

or

$K = \frac{\phi_{21}}{\phi_2} = \frac{\text{Mutual flux}}{\text{total flux}}$

→ The value of "K" varies with 0 to 1.

i) if $K=0$ then
Mutual flux $= 0$
or ϕ_{12} or $\phi_{21} = 0$

→ ~~The~~ value of it is loosely coupled or isolated couple.

ii) if $K=1$ then
Maximum mutual flux.

→ it is tightly coupled circuit

$M = \frac{N_1 \phi_{21}}{I_2}$ (for coil 1) ~~~ (i)

$M = \frac{N_2 \phi_{12}}{I_1}$ (for coil 2) ~~~ (ii)

Multiply eq^n (i) & (ii)

$\rightarrow m^2 = \frac{N_1 \phi_{21}}{I_2} \times \frac{N_2 \phi_{12}}{I_1} \times \frac{\phi_1 \phi_2}{\phi_1 \phi_2}$

$m^2 = \frac{N_1 \phi_1}{I_1} \times \frac{N_2 \phi_2}{I_2} \times \frac{\phi_{12}}{\phi_1} \times \frac{\phi_{21}}{\phi_2}$

$m^2 = L_1 \times L_2 \times K \times K$

$m^2 = L_1 L_2 K^2$

$\boxed{m = K\sqrt{L_1 L_2}}$

$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$

$0 < K < 1$

$\Rightarrow 0 < \frac{M}{\sqrt{L_1 L_2}} < 1$

$\Rightarrow 0 < M < \sqrt{L_1 L_2}$

For maximum mutual inductance

$K = 1$

$\boxed{M = \sqrt{L_1 L_2}}$

Dot Convention :-

→ Dot convention is technique, which gives the details about voltage polarity at the dotted terminal.

→ If the current enters at the dotted terminal of coil (Inductor) then it induct a voltage at another coil (Inductor) which is having -ve property at the dotted terminal.

→ If the current leaves from dotted terminal of one coil (Inductor) then it induce a volage at another coil (Inductor) which is having -ve property at the dotted terminal.

Classification of coupling :-

→ We can classify coupling in to the following catgories.

1. Electrical Coupling
2. Magnetic Coupling

1. Electrical Coupling :-

→ Electrical coupling occurs, when there exist a physical connected betn two coils (Inductors).

→ This coupling can be either of aiding type or opposing polarity type

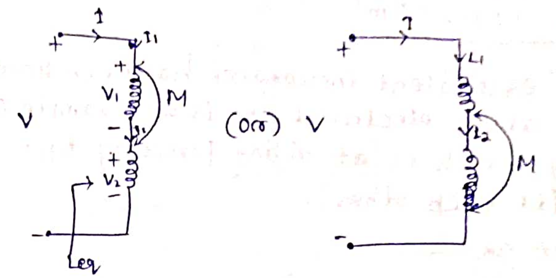
→ The electrical connection of two coil is either series type or parralled type.

→ we have discussed two connection.

1. Series type (aiding & opposing)
2. Parallel type (aiding & oppsing)

Coupling of adding patarring type :-

→ In electric circuit, which is having two inductors that are connected in series.



From figure,

$I_1 = I_2 = I$ ------ (i)

$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$

$V_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$

$V = V_1 + V_2$ -------- (ii)

$\Rightarrow V = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} + L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$ ......

$\Rightarrow V = (L_1 + L_2 + 2M) \frac{dI}{dt}$ ------ (iii) $\boxed{I_1 = I_2 = I}$

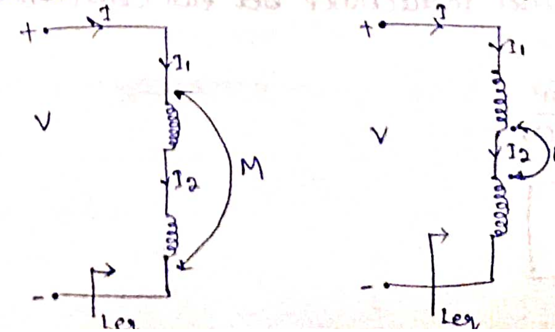
$\Rightarrow V = L_{eq} \frac{dI}{dt}$ ~~~~ (iv)

Comparing eqn (iii) & (iv)

$\boxed{L_{eq} = L_1 + L_2 + 2M}$

Opposing Polarity type :-

→ Similarly, one is dot to coil and anther is coil to dot. and coil to dot and dot to coil (in fig(2)). Then



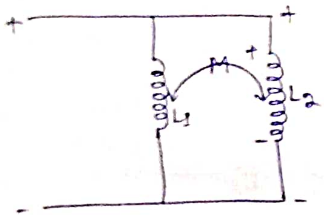
$$\boxed{L_{eq} = L_1 + L_2 - 2m}$$

→ The equivalent inductance of series combination of inductor is $\boxed{L_{eq} = L_1 + L_2 \pm 2m}$

→ In this case, the equivalent inductance has been increased by 2m. Hence the above electrical ckt is an example of electrical coupling which is at aiding property type (their fluxes aid each other).

2) <u>Parallel equivalent ckt</u> :-

→ when the two inductors are connected in parallel.



→ The total or effective inductance L_{eq} of two coils is given by.

i) fluxes adding

$$\boxed{L_{eq} = \frac{L_1L_2 - m^2}{L_1 + L_2 - 2m}}$$

ii) fluxes opposing

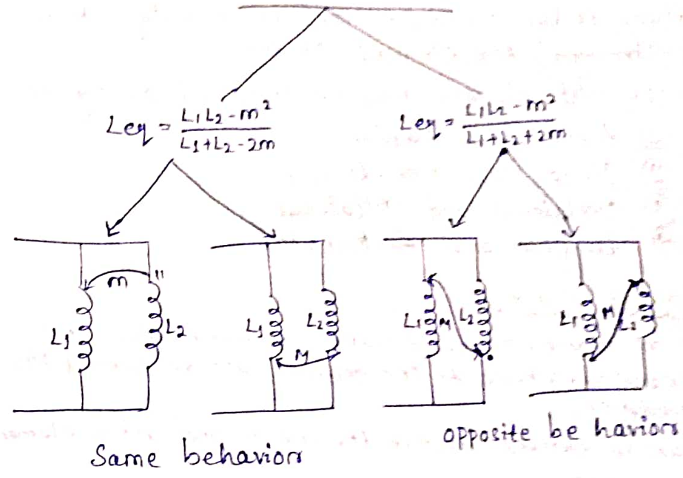
$$\boxed{L_{eq} = \frac{L_1L_2 - m^2}{L_1 + L_2 + 2m}}$$

→ if there is no mutual inductance betn two coils (i.e. m=0) then,

$$L_{eq} = \frac{L_1L_2 - (0)^2}{L_1 + L_2 + 2(0)}$$

$$\boxed{L_{eq} = \frac{L_1 \cdot L_2}{L_1 + L_2}}$$

$$L_{eq} = \frac{L_1L_2 - m^2}{L_1 + L_2 \pm 2m}$$



CHAPTER-3

Circuit Element and Analysis

→ Circuit is a network providing one or more closed path. Where as Network as an interconnection of element or devices.

→ Circuit Analysis is the process of determine voltage across (or the current through) the element of the ckt.

→ The network or ckt element may be classified as 4 groups:-

1. Active and passive
2. Linear and Non-Linear
3. Unilateral and Bilateral
4. Lumped and distributed

Active & Passive element:-

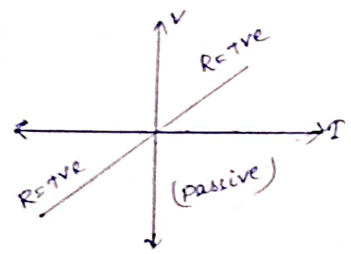
→ In case of VI plane, char. of passive element offers only positive impedance. Where as the char. of active element offers negative impedance.

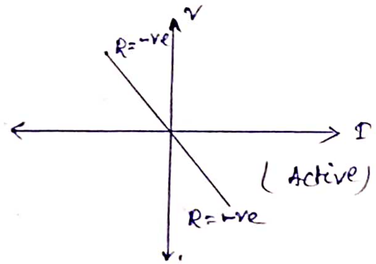
→ Normally, passive element absorb the energy and active element devoring the energy.

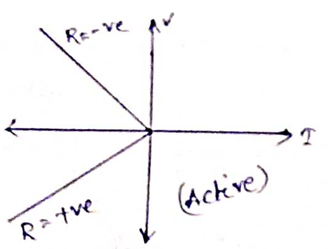
→ Active element control the flow of energy where as passive element dissipated or stored the energy.

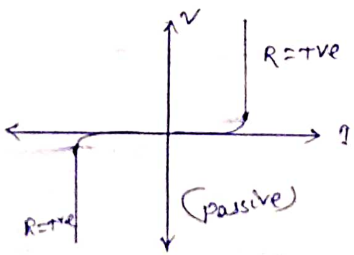
→ Example of Active Elements:- (i) voltage source (ii) Current source (iii) Generator (iv) .

→ Example of Passive Elements:- R, L, C









1st & 3rd quardant → Passive

2nd & 4th quardant → Active

(2) Liner & Non-Linear Element:-

→ The char. of Linear element always passes through the origin in the form of straight line.

→ A Linear element or network is one which satisfy the principle of superposition i.e, principle of homogeneity & additivity.

→ The element which does't satisfy the above principle is called Non-Linear element.

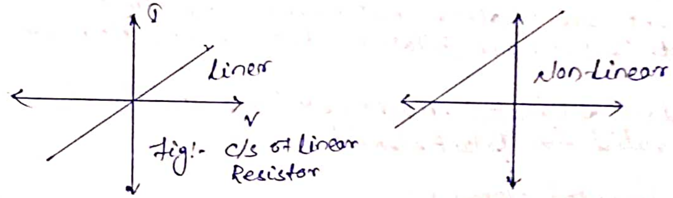


Fig:- c/s of Linear Resistor

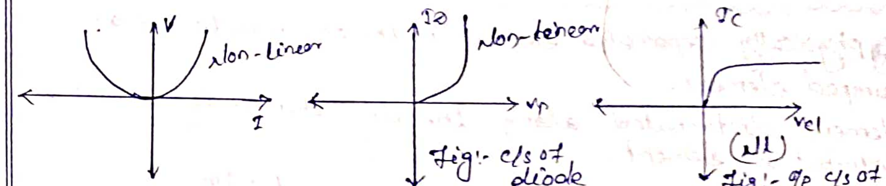


Fig:- c/s of diode

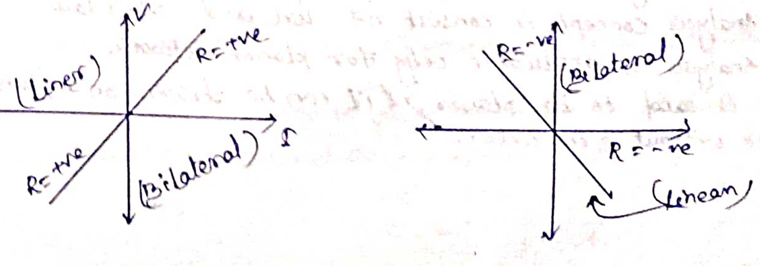
Fig:- o/p c/s of BJT

(3) Unilateral & Bilateral:-

→ In case of V-I plane, c/s of unilateral element offers different impedance in difference region, where as c/s of bilateral element offer some impedance throughout the caractarstics.

(or) A Bilateral elements, V-I plane is the same for current flowing in either direction. Where as a unilateral elements has difference relations betw voltage & current for the two possible direction of current.

→ In case of any plane, c/s of bilateral element is always symmetrical about origin.



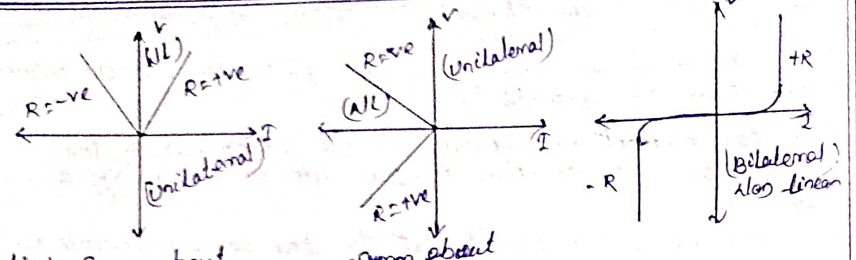
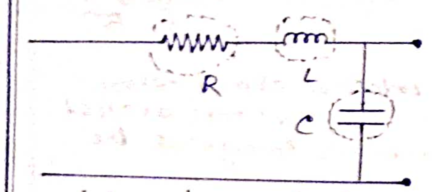


fig:- Symm about y-axis   Symm about x-axis

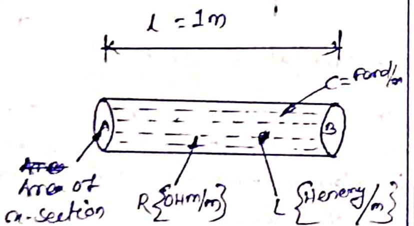
→ All Linear element are bilateral but reversis not true.
→ All n/w elements (R, L, C) are bilateral because it's c/s is symm about origin.
→ Device elements: (Diode, BJT, MOSFET etc) are unilateral.
→ Ohm's law is valid for L, B, P element.

(4) Lumped & Distributed element:-

→ physically separated element in the n/w is referred as Lumped element.
→ Element distributed along the Line then it is referred as distributed element.



(Lumped parameter R, L, C)

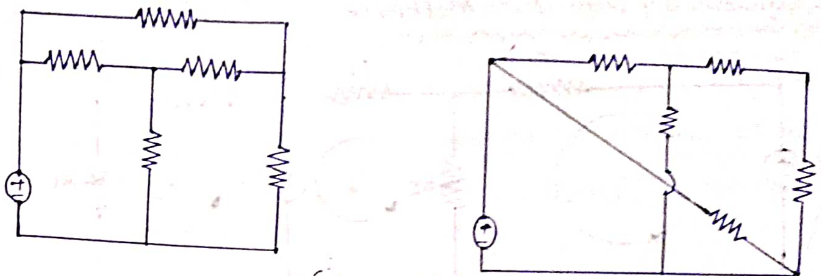


(Distributed element R, L, C)

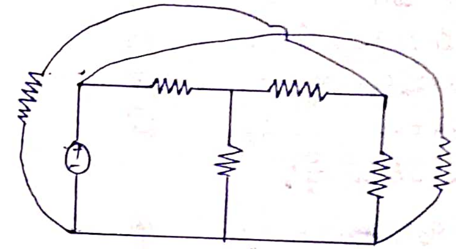
→ Ohm's law is valid for both Lumped & Distributed parameter.

Mesh Analysis:-

→ A network has a large no. of voltage source, it is useful to use mesh analysis technic. for finding solution of a network.
→ mesh Analysis concept is consist of KVL and Ohm's law.
→ mesh Analysis is applicable only for planer network.
→ A ckt is said to be planer if it can be drawn on a plane surface without crossovers.



(planer ckt)

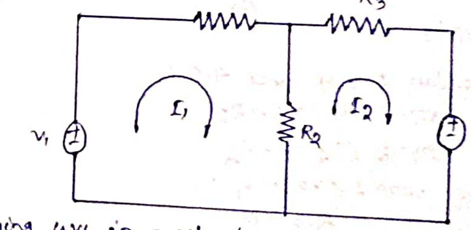


(non-planer ckt)

Condition for mesh Analysis:- (KVL + Ohm's law)

→ Temperature is constant
→ L, B, P
→ Lumped electrical N/W

Example:-



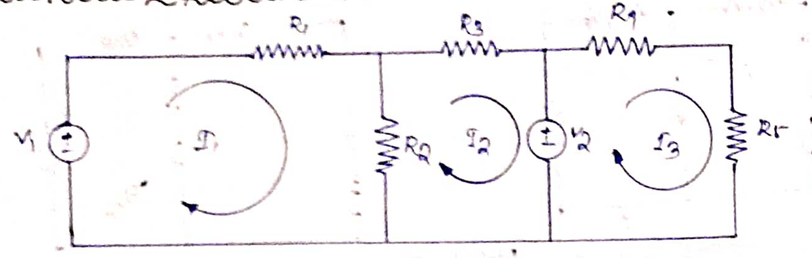
Applying KVL in mesh-1

$\Rightarrow V_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$ ---- (i)

for mesh-2 $\Rightarrow -R_2(I_2 - I_1) - I_2R_3 - V_2 = 0$ ---- (ii)

→ The Branch current may be different or same from the mesh current.
→ The no. of mesh currents is equal to the no. of mesh eqn
→ No of equations = Branches - (node-1)

Mesh Equation By Inspection Method :-



The loop equation are

$\Rightarrow I_1R_1 + R_2(I_1 - I_2) = V_1$ ------ (1)

$\Rightarrow R_2(I_2 - I_1) + I_2R_3 = -V_2$ ------ (2)

$\Rightarrow R_4I_3 + R_5I_3 = V_2$ ------ (3)

Rearranging the above eq^n

$\Rightarrow (R_1+R_2)I_1 - R_2I_2 = V_1$ ------ (4)

$\Rightarrow -R_2I_1 + (R_2+R_3)I_2 = -V_2$ ------ (5)

$\Rightarrow (R_4+R_5)I_3 = V_2$ ------ (6)

The General mesh eq^n for the three-mesh resistive network

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a$ ------ (7)

$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b$ ------ (8)

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$ ------ (9)

By Comparing the equation 7, 8, 9 and 4, 5, 6

(1) Self resistance of Loop-1 $(R_{11}) = R_1 + R_2$

(2) mutual resistance of Loop-2 $(R_{21}) = -R_2$

(3) voltage which drives Loop-1 $(V_a) = V_1$

$R_{13} = 0$

→ Self resistance of Loop-2 $(R_{22}) = R_2 + R_3$

$R_{21} = -R_2$

$R_{23} = 0$

$V_b = -V_2$

→ Self resistance of Loop-3 $(R_{33}) = R_4 + R_5$

$R_{31} = 0$

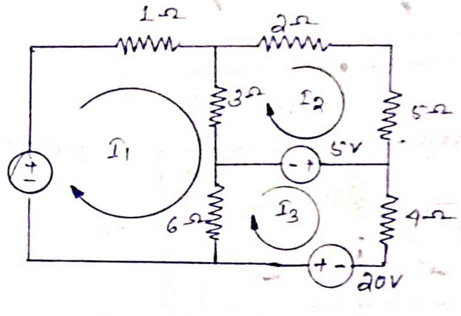
$R_{32} = 0$

$V_c = V_2$

→ If current passing through the common resistance are the same, the mutual resistance will have a positive sign and if the direction of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

→ Voltage of loop is positive sign used if the direction of the current is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the -ve sign is used.

Example :-



Solution :-

The general equation for 3-mesh network

$R_{11}I_1 + R_{12}I_2 + R_{13}I_3 = V_a$

$\pm R_{21}I_1 + R_{22}I_2 + R_{23}I_3 = V_b$

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$

Comparing the above figure,

$\Rightarrow R_{11} = 1+3+6 = 10$

$\Rightarrow R_{12} = -3\Omega$

$\Rightarrow R_{13} = -6\Omega$

$\Rightarrow V_a = +10V$

$\Rightarrow 10I_1 - 3I_2 - 6I_3 = 10V$ ---- (i)

$\Rightarrow R_{21} = -3\Omega$

$R_{22} = 3+2+5 = 10\Omega$

$R_{23} = 0$

$V_b = -5V$

$\Rightarrow -3I_1 + 10I_2 = -5V$ ---- (ii)

$\Rightarrow R_{31} = 6\Omega$

$R_{32} = 0$

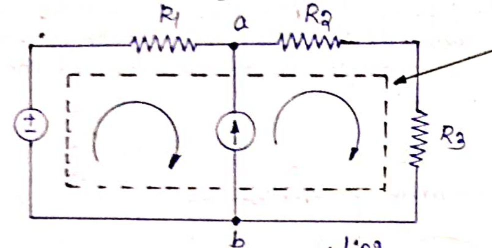
$R_{33} = 6+4 = 10\Omega$

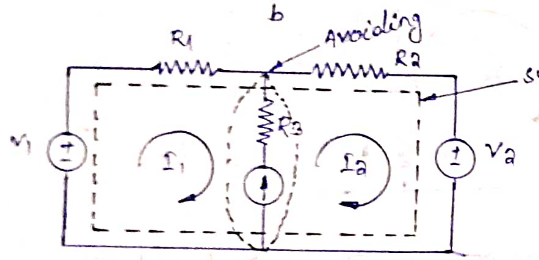
$V_c = (20+5) = 25V$

$\Rightarrow -6I_1 + 10I_3 = 25V$ ---- (iii)

Concept of Supermesh :-

If a current source (Independent or dependent) is common between two meshes, we can creat a supermesh by avoiding the current source and any elements connected in series with it.





Mesh Analysis = KVL + Ohm's law

Supermesh = KVL + Ohm's law + KCL

KVL (sm) → Applied total cut :-

$\Rightarrow -V_1 + I_1R_1 + I_2R_2 + V_2 = 0$ - - - - ①

KCL (sm) :-

$I = I_2 - I_1$ - - - - ②

$I = I_1 - I_2$ - - - - ③

By Using mesh Analysis :-

For mesh -1

$\Rightarrow -V_1 + I_1R_1 + V = 0$ - - - - ①

For mesh -2

$\Rightarrow -V_0 + I_2R_2 + V_2 = 0$ - - - - ②

Add this above two equation :-

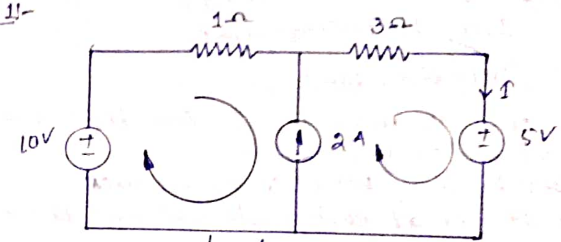
$\Rightarrow -V_1 + I_1R_1 + I_2R_2 + V_2 = 0$ - - - - ③

↳ supermesh (kvl) eqn

KCL :-

$I = I_2 - I_1$ - - - - ④

Examples 1:-



Power due to 5V is ___ W.

Soln

KVL (sm)

$\Rightarrow -10V + 1 \times I_1 + 3I_2 + 5V = 0$

$\Rightarrow I_1 + 3I_2 = 5$ - - - - (i)

KCL (sm)

$\Rightarrow 2 = I_2 - I_1$ - - - - (ii)

Adding eqn (1) x (2)

$\Rightarrow 5 = \cancel{I_1} + 3I_2$

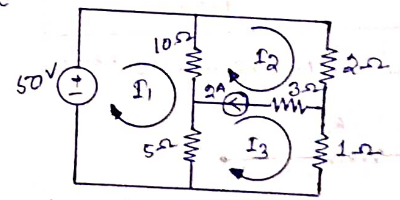
$\Rightarrow 2 = -\cancel{I_1} + I_2$

$7 = 4I_2$

$\Rightarrow I_2 = \frac{7}{4} A = I$

$P_{5V} = 5 \times \frac{7}{4} = \frac{35}{4}$ W Ans

Example -2



KVL [Loop-1/mesh 1]

$\Rightarrow -50 + (I_1 - I_2)10 + (I_1 - I_3)5 = 0$

$\Rightarrow (I_1 - I_2)10 + (I_1 - I_3)5 = 50$ - - - - ①

KVL (sm)

$\Rightarrow (I_2 - I_1)10 + 2I_2 + I_3 + (I_3 - I_1)5 = 0$

$\Rightarrow -15I_1 + 12I_2 + 6I_3 = 0$ - - - - (ii)

KCL

$I_2 - I_3 = 2A$ - - - - (iii)

Solving the aboving eqn

$I_1 = 19.99A$, $I_2 = 17.33A$, $I_3 = 15.33A$

→ The current in the 5Ω resistor = $I_1 - I_3$

$I_{5\Omega} = 19.99 - 15.33 = 4.66\,A$

Nodal Analysis :- (KCL + ohm's Law)

→ Nodal Analysis is a technique we to find the voltage at various nodes of an electric circuit.

→ This can be done by using KCL at various nodes.

→ The application of KCL at each node will gives, the nodal equation.

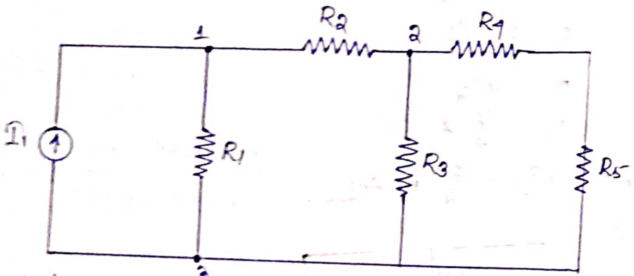
→ In a N-node circuit, one of the node is chosen as reference or datum node, then it is possible to write N-1 nodal equation by assuming N-1 node voltage.

Note:-

(i) The node voltage is the voltage of a given node w.r.t one particular node, called the reference node, which we assume at zero potential.

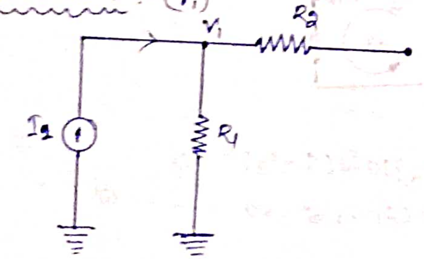
(ii) A Node is a point in an electric ckt at which current divides.

Example



→ Node-3 is assumed as the reference node

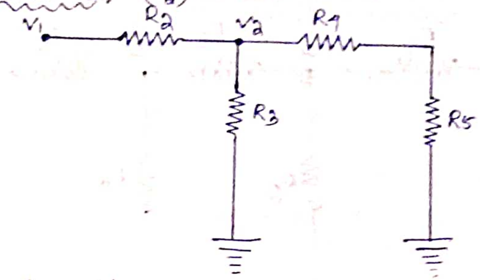
Voltage at Node-1 :- (V_1)



$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$\Rightarrow I_1 = V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right] \quad \cdots (i)$$

Voltage at Node-2 :- (V_2)



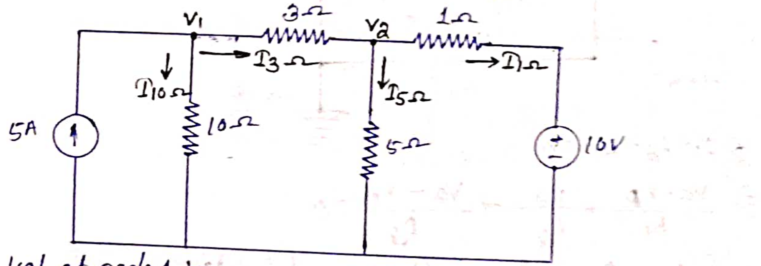
$$\Rightarrow \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

$$\Rightarrow -V_1\left[\frac{1}{R_2}\right] + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right] = 0$$

→ From the above two eqⁿ, we can find the voltage at each node.

Example

Write the node voltage equations and determine the current in each branch for the network shown in figure.



Appling KCL at node 1 :-

$$\Rightarrow \frac{V_1}{10} + \frac{V_1 - V_2}{3} = 5$$

$$\Rightarrow V_1\left[\frac{1}{10} + \frac{1}{3}\right] - V_2\left[\frac{1}{3}\right] = 5 \quad \cdots (i)$$

Appling KCL at node 2 :-

$$\Rightarrow \frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$\Rightarrow -V_1\left[\frac{1}{3}\right] + V_2\left[\frac{1}{3} + \frac{1}{5} + 1\right] = 10 \quad \cdots (ii)$$

Form solving this equation (i) and (ii) we have

$V_1 = 19.85V$, $V_2 = 10.9V$

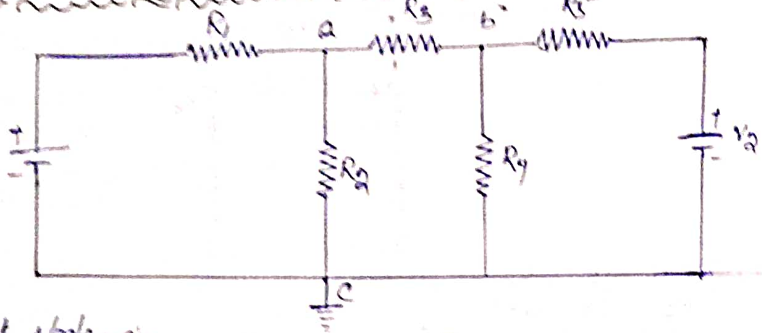
→ $I_{10\Omega} = \frac{V_1}{10} = \frac{19.85}{10} = 1.985\,A$

→ $I_{3\Omega} = \frac{V_1 - V_2}{3} = \frac{19.85 - 10.9}{3} = 2.98\,A$

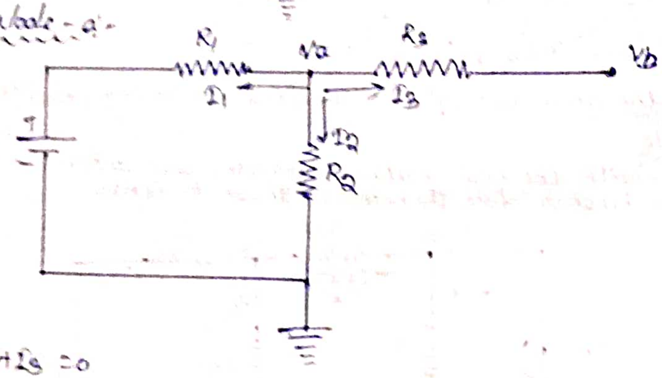
→ $I_{5\Omega} = \frac{V_2}{5} = \frac{10.9}{5} = 2.18\,A$

→ $I_{1\Omega} = \frac{V_2 - 10}{1} = 0.9A.$

Nodal Equations by Inspection method:-



KCL at Node-a:-

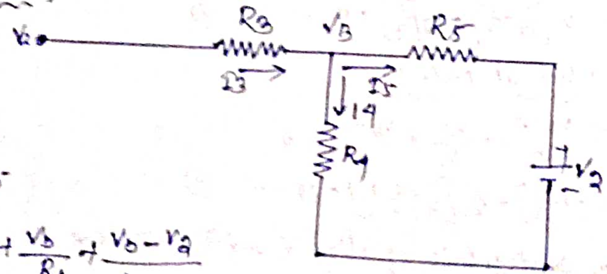


$\Rightarrow I_1 + I_2 + I_3 = 0$

$\Rightarrow \frac{V_a - V_p}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$

$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_a - \left(\frac{1}{R_3}\right) V_b = \left(\frac{1}{R_1}\right) V_1 \quad \cdots (1)$

KCL at Node-b:-



$\Rightarrow I_3 = I_4 + I_5$

$\Rightarrow \frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5}$

$\Rightarrow \left(-\frac{1}{R_3}\right) V_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) V_b = \frac{V_2}{R_5} \quad \cdots (2)$

In General Equation can be written as.

$\Rightarrow G_{aa} V_a + V_{ab} V_b = I_1 \quad \cdots (3)$

$\Rightarrow G_{ba} V_a + G_{bb} V_b = I_2 \quad \cdots (4)$

By Comparing the eq^n (1) and (2) with eq^n (3) and (4)

→ G_{aa} (self conductance) $= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \mho$

↳ Sum of the conducters connected to node 'a'

→ G_{ab} (mutual Conductance) $= \left(-\frac{1}{R_3}\right) \mho$

→ G_{ab} (mutual Conductance) $= \left(-\frac{1}{R_3}\right) \mho$

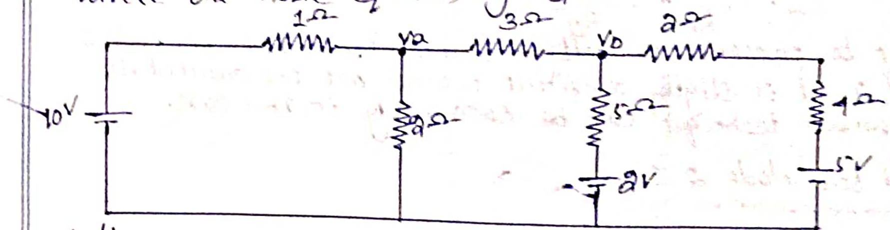
→ G_{bb} (self conductance) $= \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) \mho$

↳ Sum of Conductances connected to node 'b'.

→ I_1 and I_2 are the sum of the supom copponts at the node 'a' and the node 'b' ~~rep~~ respectively.

Example :-

Write the node equation by inspection method.



Solution:-

The general Equation are,

$\Rightarrow G_{aa} V_a + G_{ab} V_b = I_1 \quad \cdots (1)$

$\Rightarrow V_{ba} V_a + V_{bb} V_b = I_2 \quad \cdots (2)$

$G_{aa} = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3}\right) = 1.83 \mho$

$G_{bb} = \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{2}\right) = 1.03 \mho$

$G_{ab} = \left(-\frac{1}{3}\right) \mho = G_{ba} = -0.33 \mho$

$I_1 = \frac{10}{1} = 10\,A$

$I_2 = \left(\frac{2}{3} + \frac{5}{6}\right) = 1.28\,A$

Nodal Equation.

$1.83 V_a - 0.33 V_b = 10 \quad \cdots (1)$

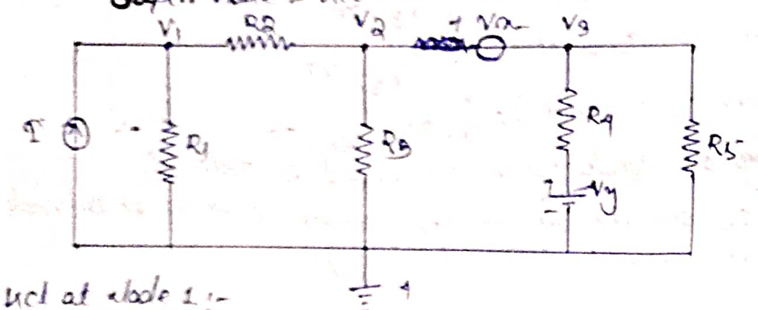
$-0.33 V_a + 1.03 V_b = 1.28 \quad \cdots (2)$

$V_a = 6.028, \quad V_b = 3.125$

Supernode Analysis :-

⇒ If the ideal voltage source (either independent or dependent) is connected betn two non-reference node then these two non reference node is formed as generalised node or super-node.

super node = KCL + ohm's law + KVL



KCL at Node 1 :-

$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

⇒ Due to presence of voltage source V_x in between node 2 and 3, it is slightly difficult to find out the current. So Supernode technique can be conventely in this case.

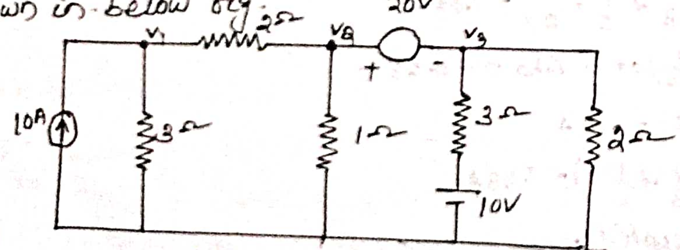
KCL for Node 2 & 3 :-

$$\Rightarrow \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_4}{R_4} + \frac{V_3}{5} = 0$$

KVL at node 2 and 3

$$\Rightarrow V_2 - V_3 = V_x$$

Ex Determine the current in the 5Ω Resistor for the ckt shown in below fig.



Solⁿ KCL at node-1

$$\Rightarrow 10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$\Rightarrow V_1\left(\frac{1}{3} + \frac{1}{2}\right) - \frac{V_2}{2} = 10 \Rightarrow 0.83V_1 - 0.5V_2 = 10 \quad \text{(1)}$$

KCL at Node 2 and Node 3 (By supernode eqⁿ)

$$\Rightarrow \frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$\Rightarrow -\frac{V_1}{2} + V_2\left(\frac{1}{2} + 1\right) + V_3\left(\frac{1}{5} + \frac{1}{2}\right) = 2$$

$$\Rightarrow -0.5V_1 + 1.5V_2 + 0.7V_3 = 2 \quad \text{(2)}$$

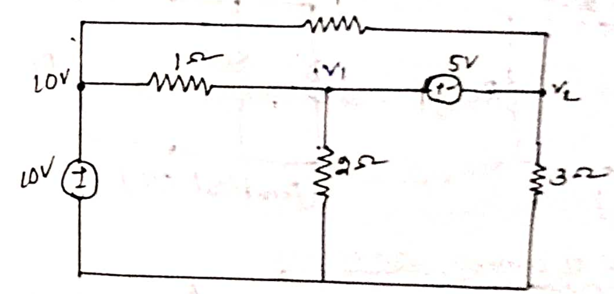
KVL at Node 2 and Node 3 :-

$$V_2 - V_3 = 20 \quad \text{(3)}$$

Solving eqⁿ (1) and (2) and (3) equ

$$V_1 = 19.03,\ V_2 = 11.59,\ V_3 = -8.40$$

Ex



Solⁿ

Node-1

$$\frac{V_1 - 10}{1} + \frac{V_1}{2} = 0$$

Node-2

$$\frac{V_2 - 10}{4} + \frac{V_2}{3} = 0$$

KVL eqⁿ $V_1 - V_2 = 5$ --- (1)

$$\frac{V_1 - 10}{1} + \frac{V_1}{2} + \frac{V_2 - 10}{4} + \frac{V_2}{3} = 0$$

$$\Rightarrow V_1\left(1 + \frac{1}{2}\right) + V_2\left(\frac{1}{3} + \frac{1}{4}\right) - 10\left(\frac{4+1}{4}\right) = 0$$

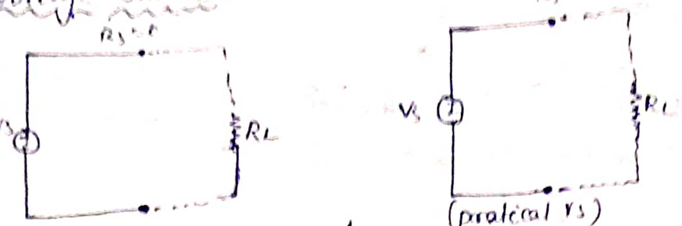
$$\Rightarrow 1.5V + 0.58V_2 = 12.5 \quad \text{(2)}$$

Solving eqⁿ (1) and (2)

$$V_1 = 7.40,\ V_2 = 2.40$$

Source transformation technique :-

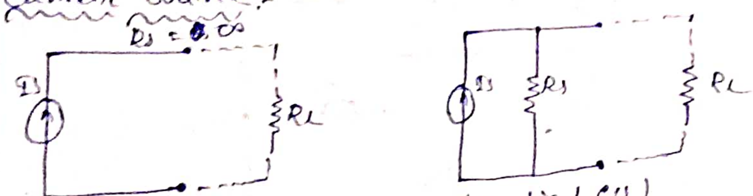
Voltage source :-



(Ideal V_s) (practical V_s)

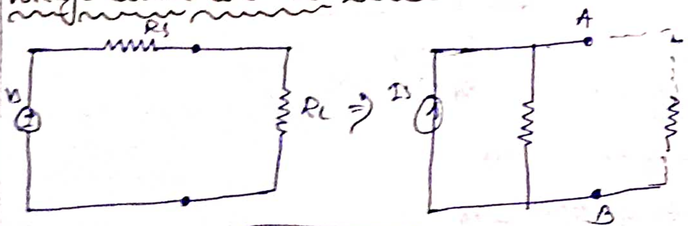
$\Rightarrow \boxed{I_s = \frac{V_s}{R_L}}$ $\boxed{I_s = \frac{V_s}{R_s + R_L}}$

Current source :-



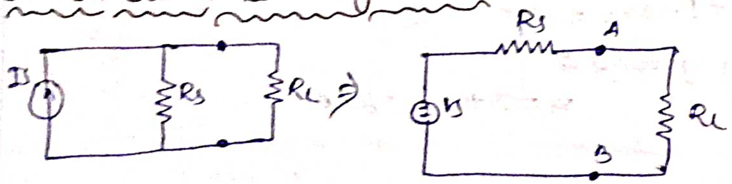
(Ideal I_s) (Practical CS)

Voltage source to current source :-



$\boxed{I_s = \frac{V_s}{R_s}}$

Current source to voltage source :-



$V_s = I_s R_s$

$\Rightarrow$ In source transformation direction of current will be from -ve terminal to '+ve' terminal of voltage source.

$\Rightarrow$ Super source transformation is valid for both independent as well as dependent practical source.

$\Rightarrow$ Source transformation is not valid for ideal voltage source and Ideal current source.

Dependent Source :-

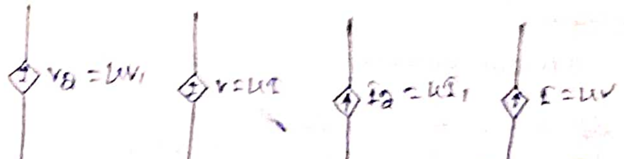
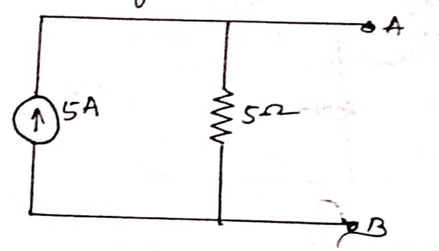


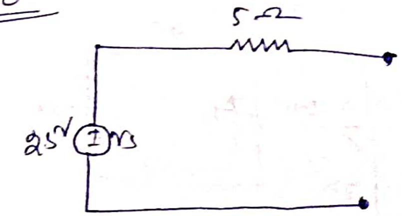
fig:- VCVS fig:- CCVS fig:- CCCS fig:- VCCS

Example :-

Determine the equivalent voltage source for the current source shown in fig.



Sol^n



$V_s = I_s R_s$

CHAPTER-4

Network theorems

⇒ Network theorems provides alternative approach for calculation of current and voltage in the network.

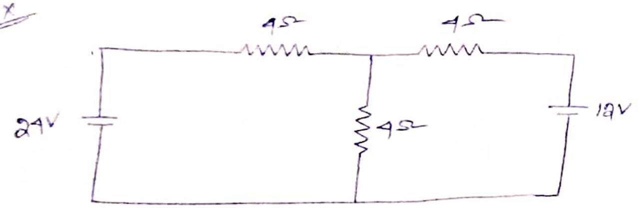
⇒ We will discussed four network theorems.

1) Superposition Theorems
2) Thevenins Theorems
3) Norton's Theorems
4) Maximum power transfer theorems.

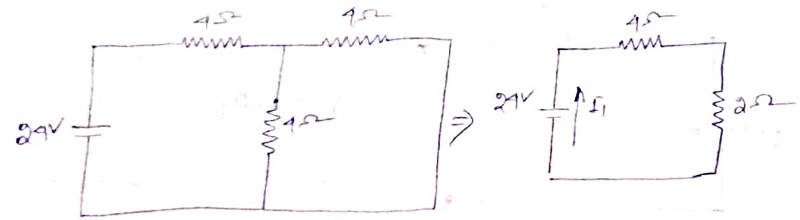
Superposition Theorem :-

⇒ It states that, In a linear network containing more than one source. If current in any branch or the potential different across any two points can be found by considering each source seperatly and then by adding their individual effect. While considering each source, the other sources are replaced by their internal resistances. If the value of internal resistance of the sources are not given, the voltage sources (Ideal) are short circuited and the current source (Ideal) are open circuited.

EX

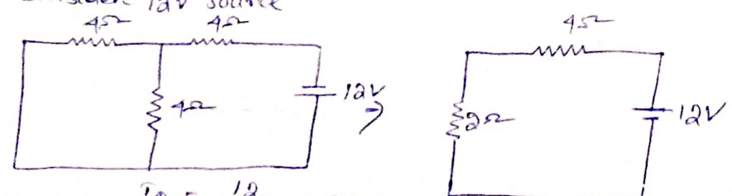


Step-1 Consider 24V sources



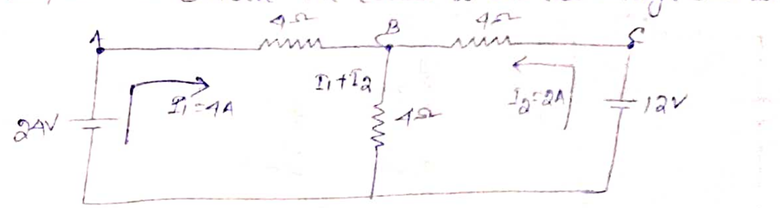
$I_1 = \frac{24}{4+2} = 4\,Amp$

Step 2 Consider 12V source



$I_2 = \frac{12}{4+2} = 2\,Amp$

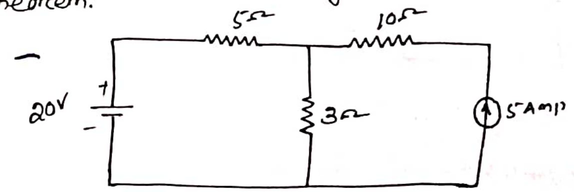
Now, we will Combine the effect of the two voltage sources.



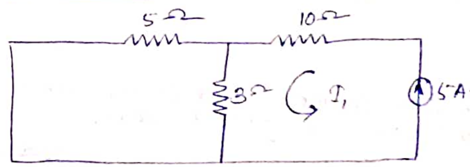
Current supplied by 24V source = 4 Amp
Current through the branch AB = 2Amp
Current supplied by the 12V source = 2Amp
Current through branch CB = 2Amp
Current through branch BD = 6 Amp

Example

Calculate the current through the 3Ω resistor by using superposition Theorem.

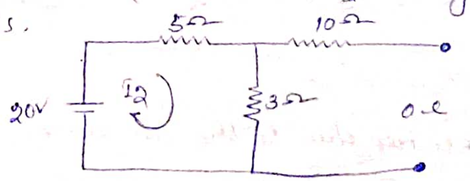


Sol^n^ The current (I_1) due to the (5Amp) source with the 20V sources short circuited is



$I_1 = 5 \times \frac{5}{3+5} = 3.125\,Amp$

The current (I_2) due to the 20V voltage source with 5A source open cut is,

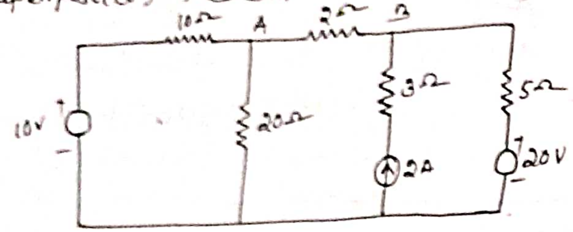


$I_2 = \frac{20}{5+3} = 2.5\,Amp$

→ The total current passing through the 3Ω resistor

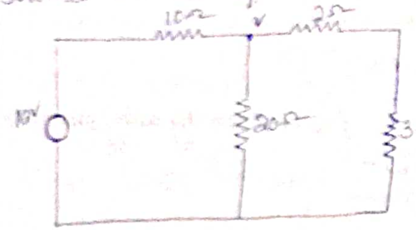
∴ $I = (2.5 + 3.125) = 5.625\,Amp.$

Ex Calculate the ~~current through~~ voltage across the 2Ω resistor by using Superposition Theorem.



Sol^n step-1

Voltage across 2Ω resistor due to the 10V source while other sources are set equal to zero.



Assuming a voltage 'V' at the node point 'A'

KCL

$\Rightarrow \frac{V-10}{10} + \frac{V}{20} + \frac{V}{7} = 0$

$\Rightarrow V(0.1 + 0.5 + 0.143) = 1$

$\Rightarrow V = 8.41V$

→ voltage across the 2Ω resistor due to the 10V source is

$V_{2\Omega} = \frac{V}{7} \times 2 = 0.97V.$

Step-2

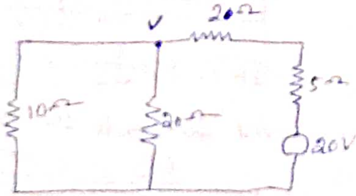
voltage across the 2Ω resistor due to the 20V source, while the other sources are set equal to zero.

KCL

$\Rightarrow \frac{V-20}{7} + \frac{V}{20} + \frac{V}{10} = 0$

$\Rightarrow V(0.143 + 0.05 + 0.1) = 2.86$

$\Rightarrow V = \frac{2.86}{0.293} = 9.76V$



⇒ The voltage across 2Ω resistor due to the 20V source is

$V_2 = \left(\frac{V-20}{7}\right) \times 2 = -2.92V$

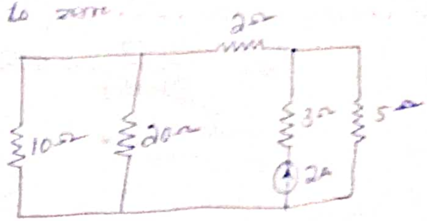
Step-3

Voltage across the 2Ω resistor due to the 2A current source while other source are set equal to zero.

The current in the 2Ω resistor,

$= 2 \times \frac{5}{5+8.67}$

$= \frac{10}{13.67} = 0.73A$



⇒ voltage across 2Ω resistor $= 0.73 \times 2 = 1.46V$

⇒ The algebric sum of these voltage gives the total voltage across the 2Ω resistor in the network.

$V = 0.97 - 2.92 - 1.46$

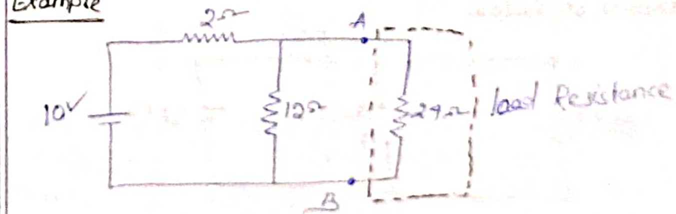
$= -3.41V$

-ve sign indicates that the voltage at 'A' is negative.

Thevenim's Theorem :-

⇒ It states that "any two terminal linear having a no of voltage & current sources and resistances can be replaced by a simple equivalent ckt consists of a single voltage source in series with a resistance where, the value of the voltage source is equal to the open ckt voltage across the two terminals of the network and resistance is equal to the equivalent resistance measured betw^n the terminals with all the energy sources are replaced by their internal resistance

Example



⇒ In the ckt, 24Ω load resistance is connected to thevenin's equivalent open ckt.

⇒ Thevenin voltage is equal to the o.c voltage across the terminal AB in the voltage across the 12Ω resistance - when the load resistance is disconnected from the ckt, the thevenin voltage

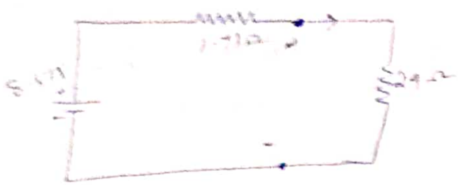
$V_{TH} = V_{oc} = 10 \times \frac{12}{12+2} = 8.57V$

⇒ The resistance into the o.c terminals is equal to the R_{TH}

$R_{TH} = \frac{12 \times 2}{12+2} = 1.71\Omega$

Now find the current passing through the 24Ω resistance and voltage across 24 due to thevenin's equivalent ckt.

$$I_{24} = \frac{8.57}{24+1.7} = 0.33A$$



Voltage across the 24Ω resistance is equal to $V_{24\Omega} = 0.33 \times 24\Omega = 7.92V$

In General method

$$I_{24\Omega} = I_T \times \frac{12}{12+24}$$

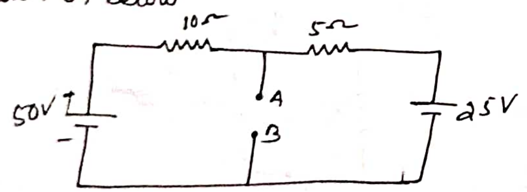
$$I_T = \frac{10}{2+(12 \parallel 24)} = \frac{10}{10} = 1\,Amp$$

$$I_{24\Omega} = 1 \times \frac{12}{12+24} = 0.33A$$

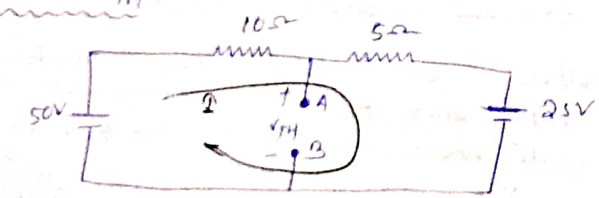
$$V_{24\Omega} = 0.33 \times 24 = 7.92\,volt$$

Example

Determine the thevenin's equavalant ckt across 'AB' for the gives ckt shown in below



To solve first V_{TH} :-



KVL

⇒ $50 - 25 = 10I + 5I$ ⇒

⇒ $\frac{25}{15} = I = 1.67\,Amp$

Voltage across $10\Omega = 10 \times 1.67 = 16.7$ volt

$V_{AB} = V_{TH} = 50 - V_{10\Omega}$

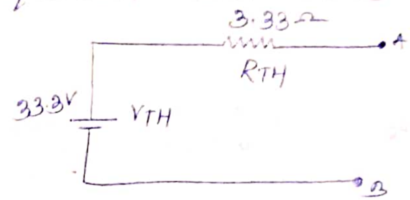
$V_{TH} = 50 - 16.7 = 33.3V$

Find R_{TH} :-

→ Two voltage source are removed & replaced with short ckt. The resistance at terminal AB is

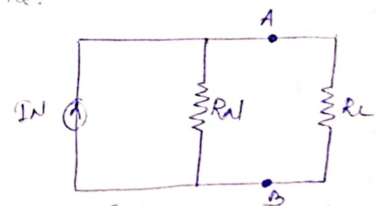
$$R_{TH} = \frac{10 \times 5}{10+5} = 3.33\Omega$$

The Equivalent Thevenin ckt is



Norton Theorem :-

→ It state that, "Any two terminal Linear network with current sources, voltage sources and resistance can be replaced by an equivalent ckt consisting of a current source in parallel with a resistance.



(Norton Equivalent ckt)

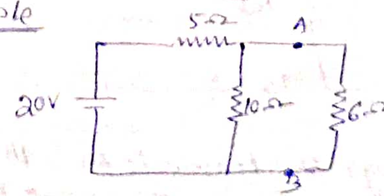
→ Norton current is referred as s.c current across the load terminal. That mean voltage across load terminal is zero.

$(I_N = I_{sc})$

→ Norton resistance is same as thevenin resistance.

$\boxed{R_N = R_{TH}}$

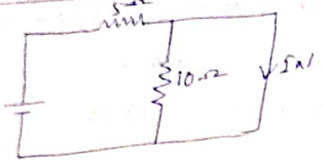
Example



For I_N

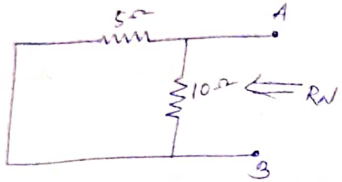
$I_N = \frac{20}{5}$

$I_N = 4$

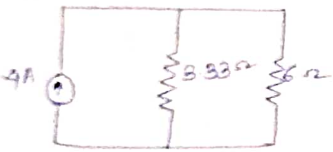


For R_N

$R_N = \frac{5 \times 10}{5+10} = 3.33\,\Omega$



Norton Equivalent ckt



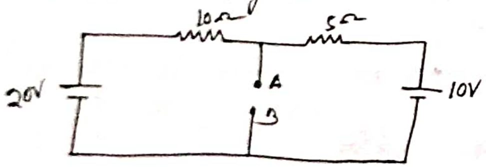
$I_{6\Omega} = 4 \times \frac{3.33}{6+3.33} = 1.43$ Amp

⇒ voltage across the 6Ω resistor

$= 1.43 \times 6 = 8.58$ volt

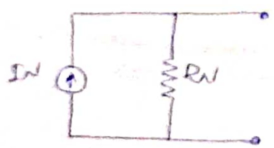
Example

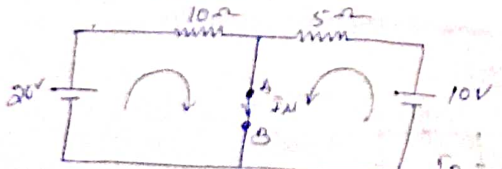
Determine Norton's equivalent ckt at terminal AB for the ckt shown in below figure.



Sol^n:-

Norton's equivalent ckt





$I_1 = \frac{20}{10} = 2$ Amp

$I_2 = \frac{10}{5} = 2$ Amp

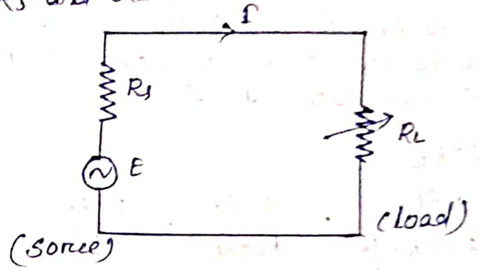
> The current passing through the terminals AB is 4A.

$R_N = \frac{10 \times 5}{10+5} = 3.33\,\Omega$

Maximum Power Transfer Theorem:-

⇒ The power is supplied from the source of the load.

⇒ Let Assume the internal resistance or source resistance of the source be R_s and the load resistance be R_L. The ckt will be



⇒ Form mpt, we find out at at what value of load (load resistance) maximum power will be transferred form the source to the load.

⇒ The current flowing in the ckt is given as follows:-

$$I = \frac{E}{R_s + R_L}$$

⇒ Power delivered is equal to power consumed assuming no line loss. Power delivered (P) is expressed as

$$P = I^2 R_L = \left(\frac{E}{R_s+R_L}\right)^2 R_L = \frac{E^2 R_L}{(R_s+R_L)^2}$$

⇒ To determine the value of R_L at which p will be maximum, we differentiate p with respect to R_L and egate to zero.

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{E^2 R_L}{(R_s+R_L)^2}\right] = 0$$

$\Rightarrow \frac{d}{d_L} E^2 [R_L (R_s+R_L)^{-2}] = 0$

$\Rightarrow E^2 [1 (R_s+R_L)^{-2} + R_L(-2)(R_s+R_L)^{-3}] = 0$

$\Rightarrow E^2 \left[\frac{1}{(R_s+R_L)^2} - \frac{2R_L}{(R_s+R_L)^3}\right] = 0$

$\Rightarrow (R_s + R_L) - 2R_L = 0$

$\Rightarrow R_s - R_L = 0$

$\Rightarrow \boxed{R_s = R_L}$

⇒ From above result shows that the maximum power will be transferred from the source to the load when the value of load resistance becomes equal to the source resistance of the source.

⇒ Maximum power is transferred to the load, when load resistance is equal to the source resistance. So, the value of maximum power is calculated as follows

$\Rightarrow P_{(max)} = I^2 R_L$

$\Rightarrow P_{(max)} = \frac{E^2 R_L}{(R_s+R_L)^2} = \frac{E^2 R_L}{(R_L+R_L)^2} = \frac{E^2 R_L}{4R_L^2}$

$\Rightarrow \boxed{P_{(max)} = \frac{E^2}{4R_L}}$ (where $R_s = R_L$)

⇒ Input delivering power is $(P_{in}) = E \times I$

$\Rightarrow P_{in} = E \times \frac{E}{R_s+R_L}$

$\Rightarrow \boxed{P_{in} = \frac{E^2}{R_s+R_L}}$

For maximum power delivering

$P_{in(max)} = \frac{E^2}{R_s+R_L}$ | when $R_s = R_L$

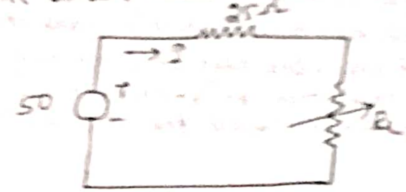
$\boxed{P_{in(max)} = \frac{E^2}{2R_L}}$

Percentage efficiency $(\% \eta) = \frac{P_{L(max)}}{P_{in(max)}} \times 100$

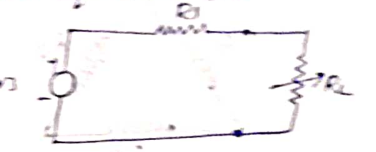
$= \frac{E^2/4R_L}{E^2/2R_L} \times 100$

$\boxed{\% \eta = 50\%}$

Example:- Determine the value of load resistance when the load resistance draws maximum power. Also find the value of the max power.



Solⁿ MPT Equivalent ckt



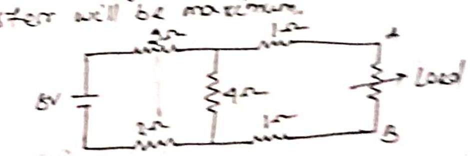
→ Source delivers the maximum power when load resistance is equal to the source resistance.

$R_S = 25\Omega$ So, $R_S = R_L = 25\Omega$

$I = \frac{50}{25+R_L} = \frac{50}{50} = 1$ Amp

$P_{(max)} = I^2 R_L = (1)^2 \times 25 = 25W$.

Ex:- A 8V battery is supplying power through a network to a load R_L as shown in fig. Calculate the value of R_L at which the power transfer will be maximum.



Solⁿ The ckt is converted into a Thevenin's equivalent ckt across terminal AB

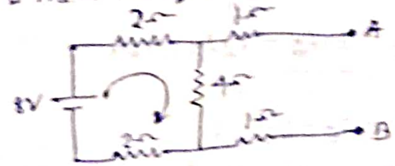
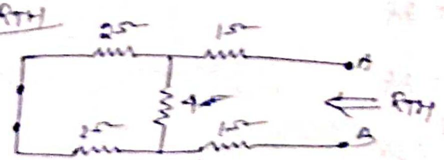


Fig - V_{TH}

$I = \frac{8}{2+4+2} = 1A$

$V_{TH} = V_{AB} = 4 \times 1 = 4V$

For R_{TH}



$R_{TH} = 2+1+1 = 4\Omega$

STAR-DELTA TRANSFORMATION :-

⇒ This transformation technique is useful in solving complex network.

⇒ Any three ckt elements, i.e. resistive, inductive, capacitive may be connected in two different ways. One way of connecting these elements is called the star connection or Y-connection. The other way of connecting these elements is called the delta(Δ) connection.

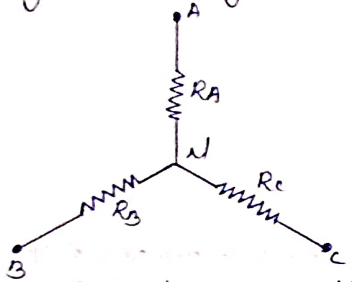


fig:- Star Y connection

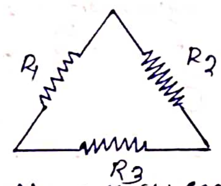


fig:- Delta(Δ) connection

⇒ When the resistor are neither in series nor in parallel connection, these type of ckt can be simplified by using 3-terminal equivalent network (Y or Δ connection).

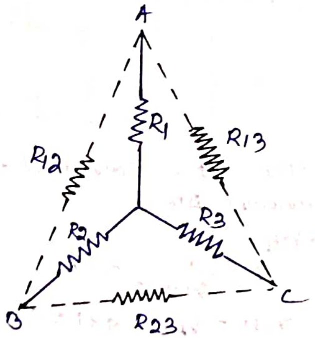
⇒ In case of star-delta conversion, for the same impendence star to delta conversion increases the impendance by factor of 3.

Star to Delta Conversion :-

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_{13} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$



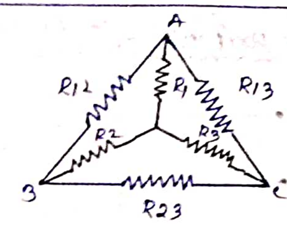
If, $R_1 = R_2 = R_3 = R$.

$$R_{12} = \frac{R^2 + R^2 + R^2}{R} = \frac{3R^2}{R} = 3R$$

$$R_{13} = \frac{R^2 + R^2 + R^2}{R} = \frac{3R^2}{R} = 3R$$

$$R_{23} = \frac{R^2 + R^2 + R^2}{R} = \frac{3R^2}{R} = 3R$$

Delta to Star Conversion :-



$$R_1 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{13} + R_{23}}$$

$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{13} + R_{23}}$$

$$R_3 = \frac{R_{13} \cdot R_{23}}{R_{12} + R_{13} + R_{23}}$$

If, $R_{12} = R_{23} = R_{31} = R$

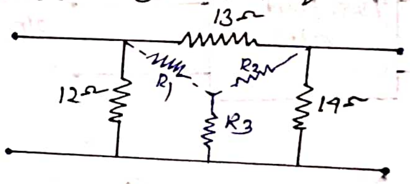
$$R_1 = \frac{R^2}{3R} = \frac{R}{3}$$

$$R_2 = \frac{R^2}{3R} = \frac{R}{3}$$

$$R_3 = \frac{R^2}{3R} = \frac{R}{3}$$

⇒ In delta to star conversion, for same impendence, decreases impendance by factor of 3.

Ex :- Obtained the star-connected equivalent for the delta connected ckt.

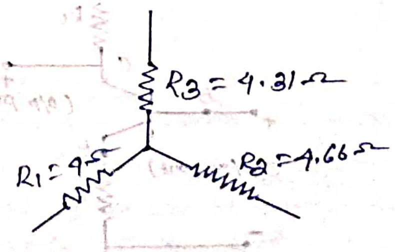


Sol^n :-

$$R_1 = \frac{12 \times 13}{12+13+14} = 4\Omega$$

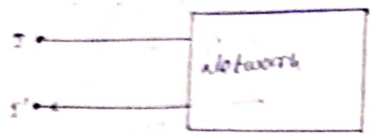
$$R_2 = \frac{13 \times 14}{12+13+14} = 4.66\Omega$$

$$R_3 = \frac{14 \times 13}{12+13+14} = 4.31\Omega$$



Two Port Network

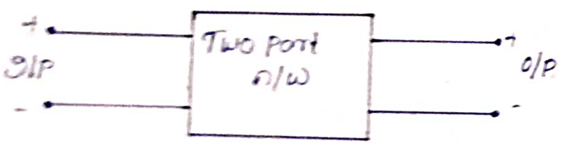
Introduction:- A pair of terminal at which a signal may either enter or leave a network is called a port.

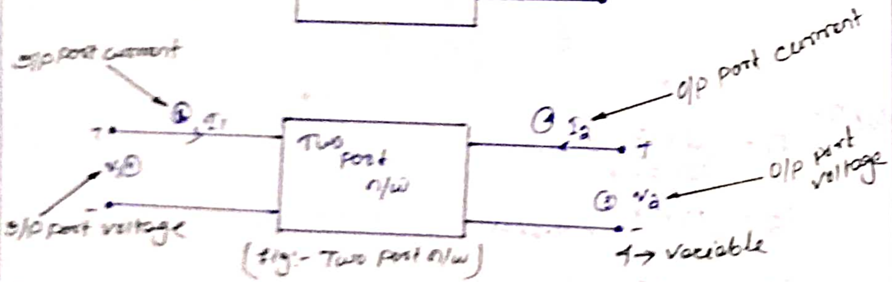


One port n/w :- The pair of terminal N/W is referred as one port n/w.

Ex:- Generator, motor etc. (Fig:- One port n/w)

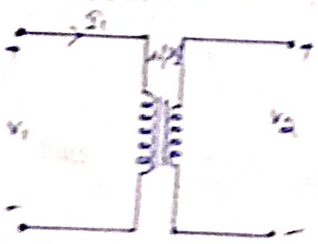
Two port n/w:- Two paired of terminal n/w is referred as two port n/w.

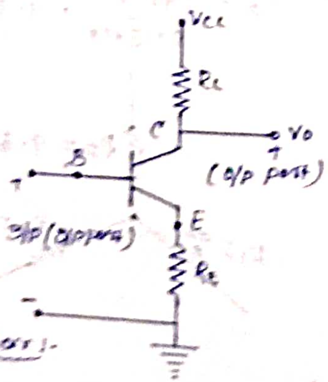




(Fig:- Two port n/w)

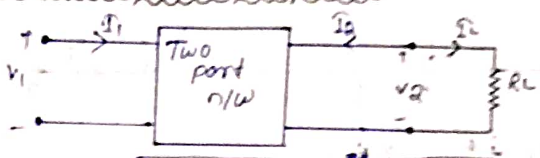
Ex:- ① Transformer:-





② Transistor:-

Representation of two port N/W:-



$\boxed{V_2 = I_L R_L}$ $\boxed{V_2 = I_2 R_2}$

$\boxed{I_L = I_2}$

I_L = load current

I_2 = O/p port current.

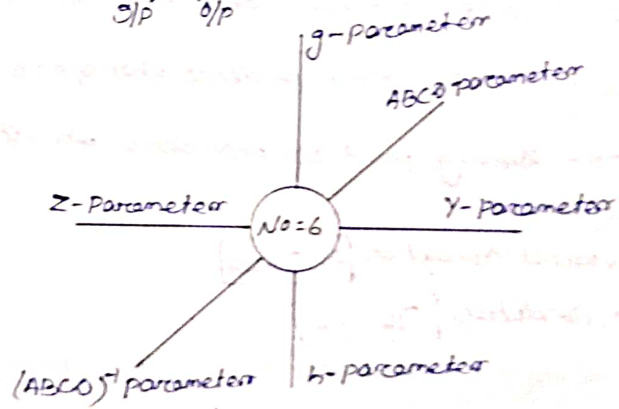
Maximum no. of possible parameter in 2 port n/w is.

$\boxed{No = {}^nC_r}$ r → I/p and o/p (2)

n → 4 variable (V_1, I_1, V_2, I_2)

$No = {}^nC_r = {}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$

I/p O/p



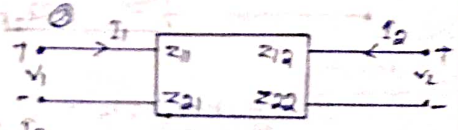
Z-Parameter (Impendence Parameter):-

(I/P KVL) $V_1 = Z_{11}I_1 + Z_{12}I_2$ ---- ①

(O/P KVL) $V_2 = Z_{21}I_1 + Z_{22}I_2$ ---- ②

$V_1 = f(I_1, I_2)$

$V_2 = f(I_1, I_2)$



Independent variables :- I_1, I_2

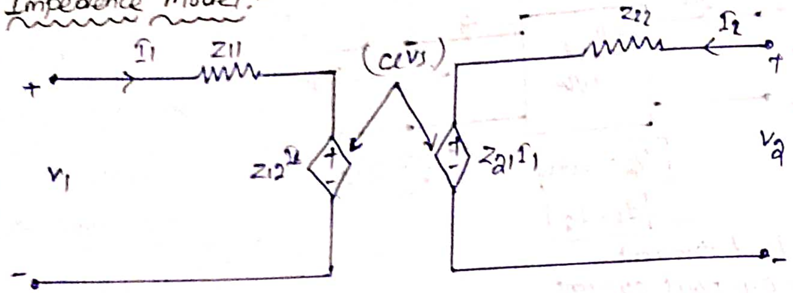
Dependent variables :- V_1, V_2

Condition for existance of z parameter:-

→ I_1 and I_2 should be independent form each other.

→ If I_1 and I_2 are dependent then z-parameter doesn't exist.

Impedence model:-



$V_1 = Z_{11}I_1 + Z_{12}I_2$ --- ①
$V_2 = Z_{21}I_1 + Z_{22}I_2$ --- ②

$Z_{11} = \frac{V_1}{I_1} \Big| I_2 = 0$ = Driving point I/P impendence when O/P = O.C

$Z_{12} = \frac{V_1}{I_2} \Big| I_1 = 0$ = Transfer I/P impendence when O/P = O.C

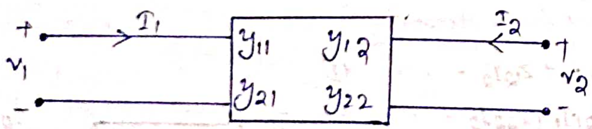
$Z_{21} = \frac{V_2}{I_1} \Big| I_2 = 0$ = Transfer O/P impendence when O/P = O.C

$Z_{22} = \frac{V_2}{I_2} \Big| I_1 = 0$ = Driving point O/P impendence when I/P = O.C

Symetrical condition:-

* For symetrical condition $\boxed{Z_{11} = Z_{22}}$
* For recipocity condition $\boxed{Z_{12} = Z_{21}}$

Y-Parameter and short ckt Parameter and Admittance parameter:-



I/P kcl :-

$I_1 = y_{11}V_1 + y_{12}V_2$ --- ①

O/P kcl :- $I_2 = y_{21}V_1 + y_{22}V_2$ --- ②

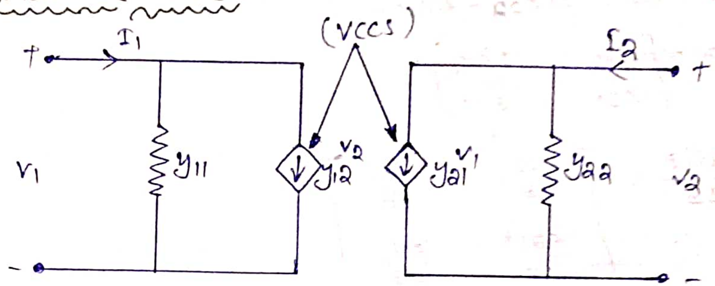
$I_1 = f(V_1, V_2)$ Independent variable = V_1, V_2

$I_2 = f(V_1, V_2)$ Dependent variable = I_1, I_2

Condition for existing of Y Parameter:-

* V_1 and V_2 should be independent at each other.
* If V_1 and V_2 are dependent then Y-parameter doesn't exist.

Admittance Model:-



$I_1 = y_{11}V_1 + y_{12}V_2$
$I_2 = y_{21}V_1 + y_{22}V_2$

Y-Parameter Element :-

$y_{11} = \frac{I_1}{V_1} \Big| V_2 = 0$ Driving point I/P admittance when O/P = S.C

$y_{12} = \frac{I_1}{V_2} \Big| V_1 = 0$ Transfer I/P admittance when I/P = S.C

$y_{21} = \frac{I_2}{V_1} \Big| V_2 = 0$ Transfer O/P admittance when O/P = S.C

$y_{22} = \frac{I_2}{V_2} \Big| V_1 = 0$ Driving point O/P admittance when I/P = SC

* Y-Parameter is referred as short-ckt parameter because in all calculation we consider either I/P is S.C or O/P is S.C.

Condition for recipocity :- $\boxed{y_{12} = y_{21}}$

Condition for symetricity :- $\boxed{y_{11} = y_{22}}$

Conversion from Z-parameter to Y-parameter:-

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V]_{2\times1} = [Z]_{2\times2}[I]_{2\times1}$$

$[I] = [V]\left[\frac{1}{Z}\right]$

$[Z]_{2\times2} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

Determinant $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

Adjoint $Z = \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$

$[Z]^{-1} = \frac{Adj\,Z}{\Delta Z} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$

$[Y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

$y_{11} = \frac{Z_{22}}{\Delta Z}$, $y_{12} = \frac{-Z_{12}}{\Delta Z}$, $y_{21} = \frac{-Z_{21}}{\Delta Z}$, $y_{22} = \frac{Z_{11}}{\Delta Z}$

Conversion form Y-parameter to Z parameter:-

$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$[I]_{2\times1} = [Y]_{2\times2}\,[V]_{2\times1}$

$[Y]_{2\times2} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

$\Delta y = y_{11}y_{22} - y_{12}y_{21}$

$Adj\,y = \begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$

$[y]^{-1} = \frac{Adj\,y}{\Delta y}$

$[y]^{-1} = \begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$

$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

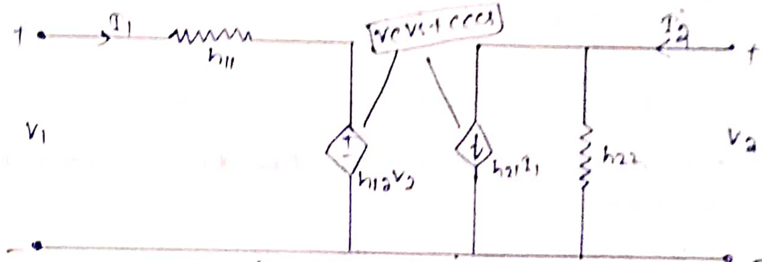
$Z_{11} = \frac{y_{22}}{\Delta y}$, $Z_{12} = \frac{-y_{12}}{\Delta y}$, $Z_{21} = \frac{-y_{21}}{\Delta y}$, $Z_{22} = \frac{y_{11}}{\Delta y}$

Hybride Parameter (H-Parameter):-

→ H-parameter is used for analysis of low frequency BJT Amplifier.

I/P KVL :- $V_1 = h_{11}I_1 + h_{12}V_2$ ... (i)

O/P KCL :- $I_2 = h_{21}I_1 + h_{22}V_2$ ... (ii)



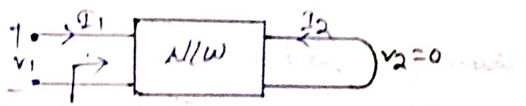
(h-Parameter)

$V_1 = f(I_1, I_2)$ — Independent variable $= I_1, V_2$

$I_2 = f(I_1, V_2)$ — Dependent variable $= V_1, I_2$

→ For existance of H-parameter I_1 & V_2 should be independent form each other.

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ Driving point i/p impedance where o/p = S.C



$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ = Reverse voltage gain when I/P = O.C

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ Forward current gain when o/p = S.C

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ Driving point o/p Admittance where I/P = O.C

$$[h]_{2\times2} = \begin{bmatrix} h_{11}(\Omega) & h_{12} \\ h_{21} & h_{22}(S) \end{bmatrix}_{2\times 2}$$

Condition:-

Reciprocity:- $h_{12} = -h_{21}$

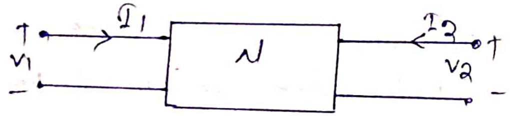
Symmetricity:- $\Delta h = 1$

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$\boxed{\Delta h = h_{11}h_{22} - h_{12}h_{21}}$$

ABCD Parameter or Transmission parameter:-

ABCD parameter is used for transient analysis of transmission line.



O/P KVL:- $V_1 = AV_2 - BI_2$ $\qquad V_1 = f(V_2, I_2)$

O/P KCL:- $I_1 = CV_2 - DI_2$ $\qquad I_1 = f(V_2, I_2)$

$\qquad I.V = V_2, I_2$

(Both KVL & KCL not exist at same time)

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

(i) $A = \frac{V_1}{V_2} \Big| I_2 = 0$ (Gain) (O.C Reverse voltage gain)

(ii) $B = \frac{-V_1}{I_2} \Big| V_2 = 0$ Ω (Transfer Impendance)

(iii) $C = \frac{I_1}{V_2} \Big| I_2 = 0$ Ω^{-1} (Transfer Admittance)

(iv) $D = \frac{-I_1}{I_2} \Big| V_2 = 0$ (Gain) (S.C reverse current gain)

Condition:-

Reciprocity:- $\Delta T = 1$

$AD - BC = 1$

Symmetricity:- $A = D$

ABCD Parameter in terms of z parameter and y-parameter:-

$V_1 = AV_2 - BI_2$; $V_1 = z_{11}I_1 + z_{12}I_2$; $I_1 = y_{11}V_1 + y_{12}V_2$

$I_1 = CV_2 - DI_2$; $V_2 = z_{21}I_1 + z_{22}I_2$; $I_2 = y_{21}V_1 + y_{22}V_2$

In Z-Parameter:-

(i) $A = \frac{V_1}{V_2} \Big| I_2 = 0$

$V_1 = z_{11}I_1 + \cancel{z_{12}I_2} \to 0$

$V_2 = z_{21}I_1 + \cancel{z_{22}I_2} \to 0$

$A = \frac{V_1}{V_2} = \frac{z_{11}\cancel{I_1}}{z_{21}\cancel{I_1}} = \frac{z_{11}}{z_{21}}$

$$\boxed{A = \frac{z_{11}}{z_{21}}}$$

(ii) $B = \frac{-V_1}{I_2} \Big| V_2 = 0$

$V_1 = z_{11}I_1 + z_{12}I_2$ —①

$\cancel{V_2}^{0} = z_{21}I_1 + z_{22}I_2$ —②

$z_{21}I_1 = -z_{22}I_2$

$I_1 = \frac{-z_{22}}{z_{21}} I_2$

Put the Eqution ①

$\Rightarrow V_1 = z_{11} \times \frac{-z_{22}}{z_{21}} \times I_2 + z_{12}I_2$

$\Rightarrow -V_1 = \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}}\right) I_2 = \frac{\Delta z}{z_{21}} \times I_2$

$B = \frac{-V_1}{I_2} = \frac{\Delta z}{z_{21}}$

(iii) $C = \frac{I_1}{V_2} \Big| I_2 = 0$

$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + \cancel{z_{22}I_2} \to 0$

$\Rightarrow \dfrac{V_2}{I_1} = z_{21}$

$\Rightarrow \dfrac{I_1}{V_2} = \dfrac{1}{z_{21}} = C$

(iv) $D = \dfrac{-I_1}{I_2} \Big| V_2 = 0$

$V_2 = z_{21}I_1 + z_{22}I_2$

$\Rightarrow -z_{21}I_1 = z_{22}I_2$

$\Rightarrow \dfrac{I_1}{I_2} = -\dfrac{z_{22}}{z_{21}}$

$\Rightarrow D = \dfrac{-I_1}{I_2} = -\left(-\dfrac{z_{22}}{z_{21}}\right) = \dfrac{z_{22}}{z_{21}}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{z_{11}}{z_{22}} & \dfrac{\Delta z}{z_{21}} \\ \dfrac{1}{z_{21}} & \dfrac{z_{22}}{z_{21}} \end{bmatrix}$$

In - Y - Parameter :-

(i) $A = \dfrac{V_1}{V_2} \Big| I_2 = 0$

$I_1 = y_{11}V_1 + y_{12}V_2$ - - - (i)

$\cancel{I_2}^{0} = y_{21}V_1 + y_{22}V_2$ - - (ii)

$\Rightarrow -y_{21}V_1 = y_{22}V_2$

$\Rightarrow \dfrac{V_1}{V_2} = \dfrac{-y_{22}}{y_{21}} = A$

(ii) $B = \dfrac{-V_1}{I_2} \Big| V_2 = 0$

$I_2 = y_{21}V_1 + \cancel{y_{22}V_2}^{0}$ . . . . (ii)

$\Rightarrow \dfrac{I_2}{V_1} = y_{21}$

$\Rightarrow \dfrac{-V_1}{I_2} = \dfrac{-1}{y_{21}}$

(iii) $C = \dfrac{I_1}{V_2} \Big| I_2 = 0$

$I_1 = y_{11}V_1 + y_{12}V_2$

$\cancel{I_2}^{0} = y_{21}V_1 + y_{22}V_2$

$\Rightarrow -y_{21}V_1 = y_{22}V_2$

$\Rightarrow I_1 = y_{11} \times \dfrac{-y_{22}}{y_{21}} V_2 + y_{12}V_2$

$\Rightarrow I_1 = \dfrac{-y_{11}y_{22}V_2 + y_{21}y_{12}V_2}{y_{21}}$

$\Rightarrow I_1 = -\left(\dfrac{y_{11}y_{22} - y_{12}y_{21}}{y_{21}}\right)V_2$

$\Rightarrow C = \dfrac{I_1}{V_2} = \dfrac{-\Delta y}{y_{21}}$

(iv) $D = \dfrac{-I_1}{I_2} \Big| V_2 = 0$

$I_1 = y_{11}V_1 + \cancel{y_{12}V_2}^{0}$

$I_2 = y_{21}V_1 + \cancel{y_{22}V_2}^{0}$

$\dfrac{I_1}{I_2} = \dfrac{y_{11}\cancel{V_1}}{y_{21}\cancel{V_1}}$

$\Rightarrow \dfrac{-I_1}{I_2} = \dfrac{-y_{11}}{y_{21}} = D$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{-y_{22}}{y_{21}} & \dfrac{-1}{z_{21}} \\ \dfrac{-\Delta y}{y_{21}} & \dfrac{-y_{11}}{y_{21}} \end{bmatrix}$$

In h - Parameter :-

(i) $A = \dfrac{V_1}{V_2} \Big| I_2 = 0$

$V_1 = h_{11}I_1 + h_{12}V_2$

$\cancel{I_2}^{0} = h_{21}I_1 + h_{22}V_2$

$\Rightarrow -h_{21}I_1 = h_{22}V_2$

$\Rightarrow I_1 = -\dfrac{h_{22}}{h_{21}}V_2$

$\Rightarrow V_1 = h_{11} \times \dfrac{-h_{22}}{h_{21}} \times V_2 + h_{12}V_2$

$\Rightarrow V_1 = \dfrac{-h_{11}h_{22}V_2 + h_{12}h_{21}V_2}{h_{21}}$

$\Rightarrow \dfrac{V_1}{V_2} = \dfrac{-(h_{11}h_{22} - h_{12}h_{21})}{h_{21}}$

$\Rightarrow \dfrac{V_1}{V_2} = \dfrac{-\Delta h}{h_{21}}$

(ii) $B = \frac{-V_1}{I_2}\Big|_{V_2=0}$

$V_1 = h_{11}I_1 + h_{12}\cancel{V_2}^{0}$

$I_2 = h_{21}I_1 + h_{22}\cancel{V_2}^{0}$

$\Rightarrow \frac{-V_1}{I_2} = -\frac{h_{11}\cancel{I_1}}{h_{21}\cancel{I_1}} = \frac{-h_{11}}{h_{21}} = B$

(iii) $C = \frac{I_1}{V_2}\Big|_{I_2=0}$

$I_2 = h_{21}I_1 + h_{22}V_2$

$\Rightarrow 0 = h_{21}I_1 + h_{22}V_2$

$\Rightarrow -h_{21}I_1 = h_{22}V_2$

$\Rightarrow \frac{I_1}{V_2} = \frac{-h_{22}}{h_{21}} = C$

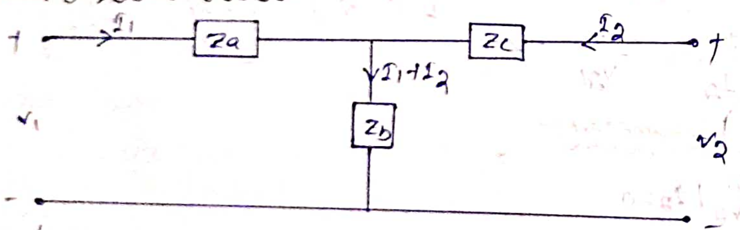
(iv) $D = \frac{-I_1}{I_2}\Big|_{V_2=0}$

$I_2 = h_{21}I_1 + h_{22}\cancel{V_2}^{0}$

$\frac{I_1}{I_2} = \frac{-1}{h_{21}} = D$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$$

T and π Representation:-



I/P kvl

$-V_1 + Z_aI_1 + Z_b(I_1+I_2) = 0$

$\Rightarrow +V_1 = +(Z_a+Z_b)I_1 + Z_bI_2$ ... (i)

O/P kvl

$-V_2 + Z_cI_2 + Z_b(I_1+I_2) = 0$

$\Rightarrow V_2 = (Z_b+Z_c)I_2 + Z_bI_1$ ... (ii)

Comparision form standrad Z parameter eq^n:-

$V_1 = (Z_a+Z_b)I_1 + Z_bI_2$

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$\boxed{Z_{11} = Z_a + Z_b}$ $\boxed{Z_{12} = Z_b}$

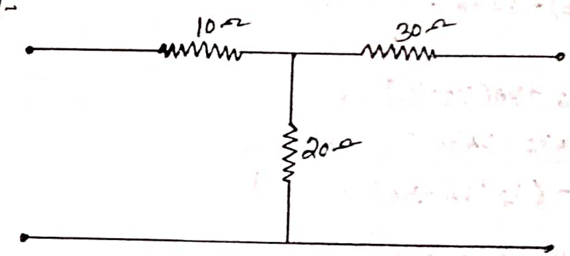
$V_2 = Z_bI_1 + (Z_b+Z_c)I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$\boxed{Z_{21} = Z_b}$ $\boxed{Z_{22} = Z_b + Z_c}$

$$[Z] = \begin{bmatrix} Z_a+Z_b & Z_b \\ Z_b & Z_b+Z_c \end{bmatrix}$$

Example:-



Sol^n

$V_1 = 10I_1 + 20(I_1+I_2)$

$\Rightarrow V_1 = 10I_1 + 20I_1 + 20I_2$

$\Rightarrow V_1 = 30I_1 + 20I_2$ —— (1)

Then

$V_2 = 30I_2 + 20(I_1+I_2)$

$\Rightarrow V_2 = 30I_2 + 20I_1 + 20I_2$

$\Rightarrow V_2 = 20I_1 + 50I_2$ —— (2)

Z-Parameter:-

$V_1 = Z_{11}I_1 + Z_{12}I_2$ ... (3)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ ... (4)

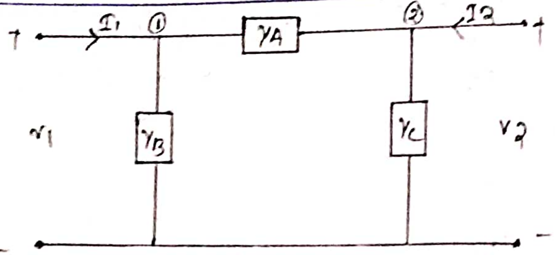
Compare eq^n (3) & (1) we get

$Z_{11} = 30$, $Z_{12} = 20$

Compare eq^n (2) & (4) we get

$Z_{21} = 20$, $Z_{22} = 50$

$$\therefore \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}$$



Put kcl at 1 :-

$-I_1 + Y_B V_1 + Y_A(V_1 - V_2) = 0$

$\Rightarrow I_1 = Y_B V_1 + Y_A V_1 - Y_A V_2$

$\Rightarrow I_1 = V_1(Y_A + Y_B) - V_2 Y_A$ . . . . (i)

Put kcl at 2 :-

$-I_2 + Y_C V_2 + Y_A(V_2 - V_1) = 0$

$\Rightarrow I_2 = Y_C V_2 + Y_A V_2 - Y_A V_1$

$\Rightarrow I_2 = -Y_A V_1 + V_2(Y_A + Y_C)$ . . (ii)

Then, $I_1 = Y_{11}V_1 + Y_{12}V_2$ . . . (iii)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ . . (iv)

Compare eqn (i) & (iii) we get

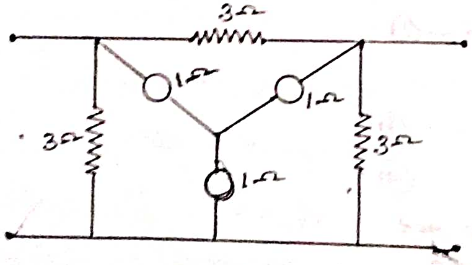
$\boxed{Y_{11} = Y_A + Y_B \ \& \ Y_{12} = -Y_A}$

Compare eqn (ii) & (iv) we get

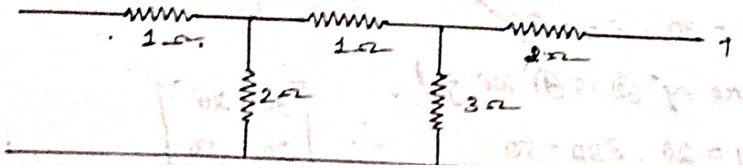
$\boxed{Y_{21} = -Y_A \ \& \ Y_{22} = Y_A + Y_C}$

$$Y = \begin{bmatrix} Y_A + Y_B & -Y_A \\ -Y_A & Y_A + Y_C \end{bmatrix}$$

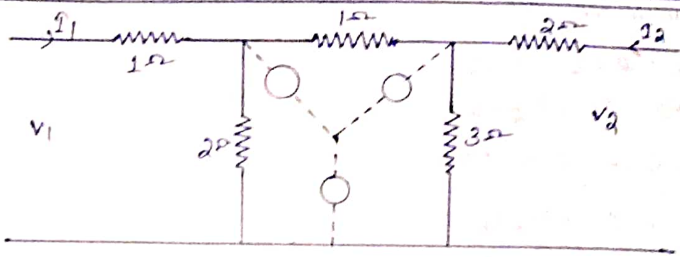
Ex



Ex



Sol^n

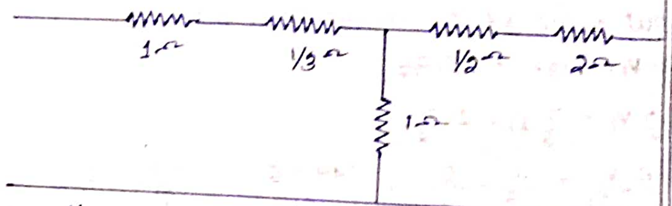


Convert π to star Connection

$R_1 = \frac{1 \times 2}{1+2+3} = \frac{2}{6} = \frac{1}{3}$

$R_2 = \frac{2 \times 3}{1+2+3} = \frac{6}{6} = 1$

$R_3 = \frac{3 \times 1}{1+2+3} = \frac{3}{6} = 1/2$



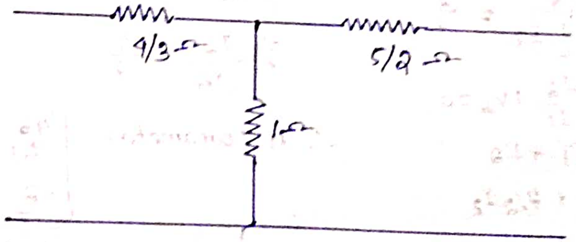
1 & 1/3 are series then

$1 + 1/3$

$= 4/3$

1/2 & 2 are series then

$1/2 + 1 = \frac{5}{2}$



① "T" parameter to 'Z' parameter

$Z_{11} = 4/3 + 1 = \frac{7}{3}$ $\qquad Z_{12} = 1$

$Z_{21} = 1$ $\qquad Z_{22} = \frac{5}{2} + 1 = \frac{7}{2}$

$$Z = \begin{bmatrix} 7/3 & 1 \\ 1 & 7/2 \end{bmatrix}$$

Z-parameter eq^n

$V_1 = 7/3 I_1 + I_2$

$V_2 = I_1 + 7/2 I_2$

② 'Z' in term of 'h' parameter:-

i) $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$h_{11} = \frac{V_1}{I_1} \Big| V_2 = 0$

$V_1 = 7/3 I_1 + I_2$

$\cancel{V_2}^{\to 0} = I_1 + 7/2 I_2$

$I_2 = -\frac{I_1 2}{7}$

put value of 'I_2' in eq^n ① we get

$\Rightarrow V_1 = 7/3 I_1 + (-I_1 2/7)$

$\Rightarrow V_1 = \frac{7}{3} I_1 - I_1 \frac{2}{7}$

$\Rightarrow \frac{V_1}{I_1} = \frac{7}{3} - \frac{2}{7} = \frac{49-6}{21} = \frac{43}{21} = h_{11}$

(ii) $h_{12} = \frac{V_1}{V_2} \Big| I_1 = 0$

$V_1 = \cancel{7/3 I_1}^{\to 0} + I_2$

$V_2 = \cancel{I_1}^{\to 0} + 7/2 I_2$

$\frac{V_1}{V_2} = \frac{I_2}{\frac{7}{2} I_2} = \frac{2}{7}$

(iii) $h_{21} = \frac{I_2}{I_1} \Big| V_2 = 0$

$V_1 = 7/3 I_1 + I_2$

$\cancel{V_2}^{\to 0} = I_1 + 7/2 I_2$

$\Rightarrow -I_1 = \frac{7}{2} I_2$

$\Rightarrow -I_1 \frac{2}{7} = I_2$

$\Rightarrow \frac{I_2}{I_1} = -\frac{2}{7}$

iv) $h_{22} = \frac{I_2}{V_2} \Big| I_1 = 0$

$V_1 = \cancel{7/3 I_1}^{\to 0} + I_2$

$V_2 = \cancel{I_1}^{\to 0} + 7/2 I_2$

$\Rightarrow V_2 = \frac{7}{2} I_2$

$\Rightarrow \frac{I_2}{V_2} = \frac{2}{7}$

$\therefore$ h parameter $= \begin{bmatrix} \frac{43}{21} & \frac{2}{7} \\ -\frac{2}{7} & \frac{2}{7} \end{bmatrix}$

Z-interm of ABCD parameter:-

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

i) $A = \frac{V_2}{V_1} \Big| I_2 = 0$

$V_1 = 7/3 I_1 + \cancel{I_2}^{\to 0}$

$V_2 = I_1 + \cancel{7/2 I_2}^{\to 0}$

$\Rightarrow \frac{V_2}{V_1} = \frac{I_1}{7/3 I_1} = \frac{3}{7}$

(ii) $B = \frac{-V_1}{I_2} \Big| V_2 = 0$

$V_1 = 7/3 I_1 + I_2$ - - ①

$\cancel{V_2}^{\to 0} = I_1 + 7/2 I_2$ — ②

$-I_1 = 7/2 I_2$

$\therefore I_1 = -7/2 I_2$

put the value of 'I_1' in eq^n ①

$\Rightarrow V_1 = 7/3(-7/2 I_2) + I_2$

$\Rightarrow V_1 = -\frac{49}{6} I_2 + I_2$

$\Rightarrow V_1 = I_2(-\frac{49}{6} + 1)$

$\Rightarrow \frac{V_1}{I_2} = \left(\frac{-49+6}{6}\right) = \frac{-43}{6}$

$\Rightarrow \frac{-V_1}{I_2} = \frac{43}{6}$

(iii) $C = \frac{I_1}{V_2} \Big| I_2 = 0$

$V_1 = 7/3 I_1 + \cancel{I_2}^{\to 0}$

$V_2 = I_1 + \cancel{7/2 I_2}^{\to 0}$

$\Rightarrow V_2 = I_1$

$\Rightarrow \frac{I_1}{V_2} = 1$

(iv) $D = \frac{-I_1}{I_2} \Big| V_2 = 0$

$V_1 = 7/3 I_1 + I_2$ - - ①

$\cancel{V_2}^{\to 0} = I_1 + 7/2 I_2$ - - ②

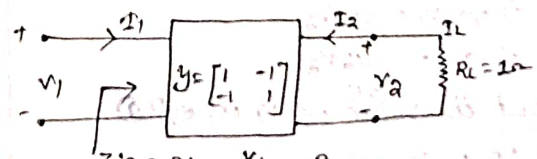
$\Rightarrow -I_1 = 7/2 I_2$

$\Rightarrow \frac{-I_1}{I_2} = 7/2$

$\therefore$ ABCD parameter matrix is

$\begin{bmatrix} 3/7 & \frac{43}{6} \\ 1 & 7/2 \end{bmatrix}$

Q.



$Z_{in} = R_{in} = \frac{V_1}{I_1} = ?$

Sol^n

$I_1 = y_{11} V_1 + y_{12} V_2$

$I_1 = V_1 - V_2$ - - ①

$I_2 = y_{21} V_1 + y_{22} V_2$

$I_2 = -V_1 + V_2$ - - - ⓪

$V_2 = -I_2 R_2 = -I_2 \times 1 = -I_2$ $\boxed{V_2 = -I_2}$ - - - (iii)

Form eq^n (2) and (1) we get

$-V_2 = -V_1 + V_2$

$\Rightarrow V_1 = 2V_2$

$\Rightarrow V_2 = \frac{V_1}{2}$

Form eq^n (1)

$\Rightarrow I_1 = V_1 - \frac{V_1}{2}$

$\Rightarrow I_1 = \frac{2V_1 - V_1}{2}$

$\Rightarrow I_1 = \frac{V_1}{2} \quad \Rightarrow \frac{V_1}{I_1} = Z_{in} = 2\Omega$

$\therefore Z_{in} = R_{in} = 2\Omega$

Conversion of "y" parameters into "h" parameters :-

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

(i) $h_{11} = \frac{V_1}{I_1} \Big| V_2 = 0$

$I_1 = y_{11}V_1 + \cancel{y_{12}V_2} \to 0$

$I_2 = y_{21}V_1 + \cancel{y_{22}V_2} \to 0$

$\Rightarrow \frac{V_1}{I_1} = \frac{1}{y_{11}}$

(ii) $h_{12} = \frac{V_1}{V_2} \Big| I_1 = 0$

$\cancel{I_1} \overset{0}{=} y_{11}V_1 + y_{12}V_2$

$\Rightarrow -y_{11}V_1 = y_{12}V_2$

$\Rightarrow \frac{V_1}{V_2} = \frac{-y_{12}}{y_{11}}$

(iii) $h_{21} = \frac{I_2}{I_1} \Big| V_2 = 0$

$I_1 = y_{11}V_1 + \cancel{y_{12}V_2} \to 0$

$I_2 = y_{21}V_1 + \cancel{y_{22}V_2} \to 0$

$\Rightarrow \frac{I_2}{I_1} = \frac{y_{21}V_1}{y_{11}V_1} = \frac{y_{21}}{y_{11}}$

(iv) $h_{22} = \frac{I_2}{V_2} \Big| I_1 = 0$

$\cancel{I_1} \overset{0}{=} y_{11}V_1 + y_{12}V_2$ --- (1)

$I_2 = y_{21}V_1 + y_{22}V_2$ --- (2)

$\Rightarrow -y_{11}V_1 = y_{12}V_2$

$\Rightarrow V_1 = -\frac{y_{12}V_2}{y_{11}}$

Put the value of V_1 in eq^n (2)

$\Rightarrow I_2 = y_{21} \cdot \frac{-y_{12}V_2}{y_{11}} + y_{22}V_2$

$\Rightarrow I_2 = \left(\frac{-y_{11}y_{21} - y_{11}y_{22}}{y_{11}}\right) + V_2$

$\Rightarrow \frac{I_2}{V_2} = \frac{-\Delta y}{y_{11}}$

$\therefore h = \begin{bmatrix} 1/y_{11} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{-\Delta y}{y_{11}} \end{bmatrix}$

Conversion of "h" parameter into "y" parameter :-

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

(i) $y_{11} = \frac{I_1}{V_1} \Big| V_2 = 0$

$\therefore V_1 = h_{11}I_1 + \cancel{h_{12}V_2} \to 0$

$I_2 = h_{21}I_1 + \cancel{h_{22}V_2} \to 0$

$\Rightarrow V_1 = h_{11}I_1$

$\Rightarrow \frac{I_1}{V_1} = \frac{1}{h_{11}}$

(ii) $y_{12} = \frac{I_1}{V_2} \Big| V_1 = 0$

$\cancel{V_1} \overset{0}{=} h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$\Rightarrow -h_{11}I_1 = h_{12}V_2$

$\Rightarrow \frac{I_1}{V_2} = -\frac{h_{12}}{h_{11}}$

(iii) $y_{21} = \frac{I_2}{V_1} \Big| V_2 = 0$

$V_1 = h_{11}I_1 + \cancel{h_{12}V_2} \to 0$

$I_2 = h_{21}I_1 + \cancel{h_{22}V_2} \to 0$

$\Rightarrow \frac{I_2}{V_1} = \frac{h_{21}I_1}{h_{11}I_1} = \frac{h_{21}}{h_{11}}$

(iv) $y_{22} = \frac{I_2}{V_2} \Big| V_1 = 0$

$\cancel{V_1} \overset{0}{=} h_{11}I_1 + h_{12}V_2$ --- (1)

$I_2 = h_{21}I_1 + h_{22}V_2$ --- (2)

$\Rightarrow -h_{11}I_1 = h_{12}V_2$

$\Rightarrow I_1 = -\frac{h_{12}V_2}{h_{11}}$

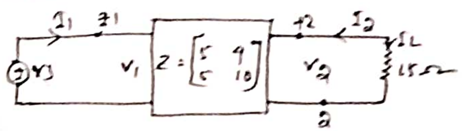
Put the value of "I_1" in the eq^n (2)

$\Rightarrow I_2 = h_{21} \cdot -\frac{h_{12}}{h_{11}}V_2 + h_{22}V_2$

$\Rightarrow I_2 = \left(\frac{-h_{12} \cdot h_{21} + h_{11} \cdot h_{22}}{h_{11}}\right) V_2$

$\Rightarrow \frac{I_2}{V_2} = \frac{\Delta h}{h_{11}}$

PROBLEM :-



$\therefore V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$\therefore V_1 = 5I_1 + 4I_2$ --- (1)

$V_2 = 5I_1 + 10I_2$ --- (2)

$V_2 = I_L \cdot R_L = -I_2 \cdot R_L$

$= -I_2 \cdot 15$

$\boxed{\therefore V_2 = -I_2 \cdot 15}$ --- (3)

put the value of eq (3) in eq (2)

$\Rightarrow -I_2 15 = 5I_1 + 10I_2$

$\Rightarrow -I_2 15 - 10I_2 = 5I_1$

$\Rightarrow I_2 = \frac{-5I_1}{25} \Rightarrow \boxed{I_2 = \frac{I_1}{5}}$

put the value of "I_2" in eq (1)

$\Rightarrow V_1 = 5I_1 - 4\frac{I_1}{5}$

$= 5I_1 - \frac{4I_1}{5}$

$= \frac{25I_1 - 4I_1}{5} = \frac{21I_1}{5} \Rightarrow \boxed{V_1 = \frac{21I_1}{5}}$

$Z_{11} = 5$

$Z_{11} = \frac{V_1}{I_1} | I_2 = 0$

$\Rightarrow 5 = \frac{25}{I_1} \Rightarrow \boxed{I_1 = 5}$

Again $Z_{12} = \frac{V_1}{I_2} | I_1 = 0$

$\Rightarrow 4 = \frac{25}{I_2} \Rightarrow \boxed{I_2 = \frac{25}{4}}$

Put the value of "I_1" and "I_2" in eq^n (2)

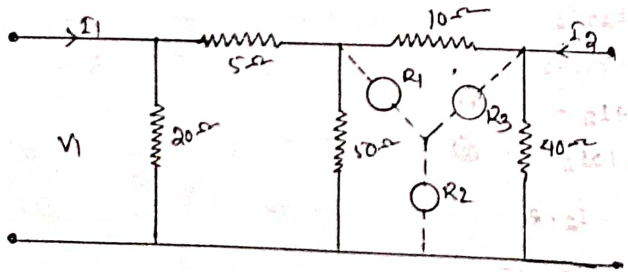
$V_2 = 5I_1 + 10I_2$

$\Rightarrow V_2 = 5 \times 5 + 10 \times \frac{25}{4}$

$= \frac{100 + 250}{4} = \frac{\cancel{350}\,175}{\cancel{4}\,2}$

$\boxed{V_2 = \frac{175}{2}}$

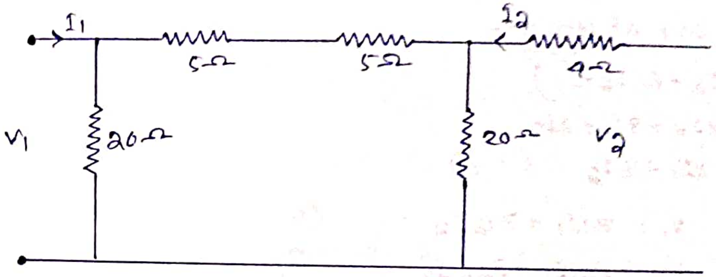
2)



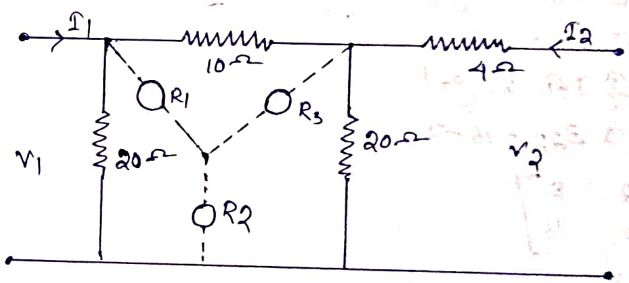
Sol^n $R_1 = \frac{50 \times 10}{50 + 40 + 10} = \frac{500}{100} = 5\Omega$

$R_2 = \frac{50 \times 40}{50 + 10 + 40} = \frac{2000}{100} = 20\Omega$

$R_3 = \frac{10 \times 40}{50 + 10 + 40} = \frac{400}{100} = 4\Omega$



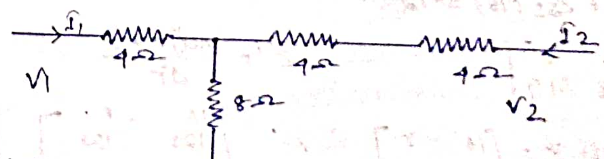
5Ω & 5Ω are connected in series so $5 + 5 = 10\Omega$



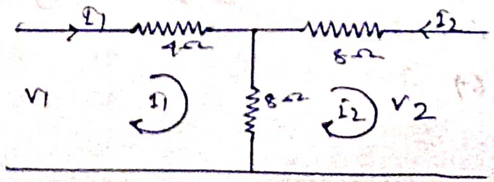
$R_1 = \frac{20 \times 10}{20 + 10 + 20} = \frac{200}{50} = 4\Omega$

$R_2 = \frac{20 \times 20}{20 + 10 + 20} = \frac{400}{50} = 8\Omega$

$R_3 = \frac{20 \times 10}{20 + 10 + 20} = \frac{200}{50} = 4\Omega$



4Ω & 4Ω are connected in series, so $4 + 4 = 8\Omega$



Applying KVL at loop 1:-

$V_1 = 4I_1 + 8(I_1 + I_2)$

$\Rightarrow V_1 = 4I_1 + 8I_1 + 8I_2$

$\Rightarrow V_1 = 12I_1 + 8I_2$ --- ①

Applying KVL at loop 2:-

$V_2 = 8I_2 + 8(I_2 + I_1)$

$\Rightarrow V_2 = 8I_2 + 8I_1 + 8I_2$

$\Rightarrow V_2 = 8I_1 + 16I_2$ --- ②

Again $V_1 = Z_{11}I_1 + Z_{12}I_2$ --- ③

$V_2 = Z_{21}I_1 + Z_{22}I_2$ --- ④

Comparing eqn ① & ③ we get

$Z_{11} = 12\Omega$ & $Z_{12} = 8\Omega$

Comparing eqn ② & ④ we get

$Z_{21} = 8\Omega$ & $Z_{22} = 16\Omega$

$\therefore Z = \begin{bmatrix} 12 & 8 \\ 8 & 16 \end{bmatrix}$

Conversion 'Z' to Y

$Z = \begin{bmatrix} 12 & 8 \\ 8 & 16 \end{bmatrix}$

Cofactor of $Z_{11}(12) = 16$

Cofactor of $Z_{12}(8) = -8$

Cofactor of $Z_{21}(8) = -8$

Cofactor of $Z_{22}(16) = -12$

$= \begin{bmatrix} 16 & 8 \\ -8 & 12 \end{bmatrix}$

$\therefore$ Adj of $Z = \begin{bmatrix} 16 & -8 \\ -8 & 12 \end{bmatrix}$

$\therefore \Delta Z = 16 \cdot 12 - (-8 \cdot -8)$

$= 192 - 64$

$= 128$

$Z^{-1} = \frac{\text{Adj of } Z}{\Delta Z}$

$= \begin{bmatrix} \frac{16}{128} & \frac{-8}{128} \\ \frac{-8}{128} & 12/128 \end{bmatrix} = \begin{bmatrix} \frac{4}{132} & \frac{-2}{132} \\ \frac{-2}{132} & 3/132 \end{bmatrix}$

Conversion Z to h:-

$V_1 = h_{11}I_1 + \cancel{h_{12}V_2}\ h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

(i) $h_{11} = \frac{V_1}{I_1} \mid V_2 = 0$

$V_1 = \cancel{h_{11}I_1}\ 12I_1 + 8I_2$ —— ①

$\cancel{V_2}^0 = 8I_1 + 16I_2$ —— ②

$-8I_1 = 16I_2$

$\Rightarrow I_2 = \frac{-8I_1}{16} = -1/2\, I_1$

put the value of "I_2" in eqn ①

$V_1 = 12I_1 + 8 \times \frac{-1}{2} I_1$

$\Rightarrow V_1 = 12I_1 - \frac{4}{1} I_1$

$\Rightarrow \frac{V_1}{I_1} = 12 - \frac{4}{1}$

$\Rightarrow \frac{V_1}{I_1} = 8$

(ii) $h_{12} = \frac{V_1}{V_2} \mid I_1 = 0$

$V_1 = \cancel{12I_1}^0 + 8I_2$

$V_2 = \cancel{8I_1}^0 + 16I_2$

$\Rightarrow \frac{V_1}{V_2} = \frac{8I_2}{16I_2} = \frac{1}{2}$

(iii) $h_{21} = \frac{I_2}{I_1} \mid V_2 = 0$

$\cancel{V_2}^0 = 8I_1 + 16I_2$

$\Rightarrow -8I_1 = 16I_2$

$\Rightarrow \frac{I_2}{I_1} = -\frac{8}{16} = -\frac{1}{2}$

(iv) $h_{22} = \frac{I_2}{V_2} \mid I_1 = 0$

$V_2 = \cancel{8I_1}^0 + 16I_2$

$\Rightarrow \frac{I_2}{V_2} = \frac{1}{16}$

$H = \begin{bmatrix} 8 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{16} \end{bmatrix}$

Conversion of Z to ABCD Parameter:-

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

(i) $A = \frac{V_1}{V_2} \mid I_2 = 0$

$V_1 = 12I_1 + \cancel{8I_2}^0$

$V_2 = 8I_1 + \cancel{16I_2}^0$

$\frac{V_1}{V_2} = \frac{12I_1}{8I_1} = \frac{12}{8}$

(ii) $B = \frac{-V_1}{I_2} \mid V_2 = 0$

$V_1 = 12I_1 + 8I_2$ —— ①

$\cancel{V_2}^0 = 8I_1 + 16I_2$ —— ②

$\Rightarrow I_1 = \frac{-16I_2}{8} = -2I_2$

put the value of "I_1" in eqn ①

$V_1 = 12 \times (-2I_2) + 8I_2$

$\Rightarrow V_1 = -24I_2 + 8I_2$

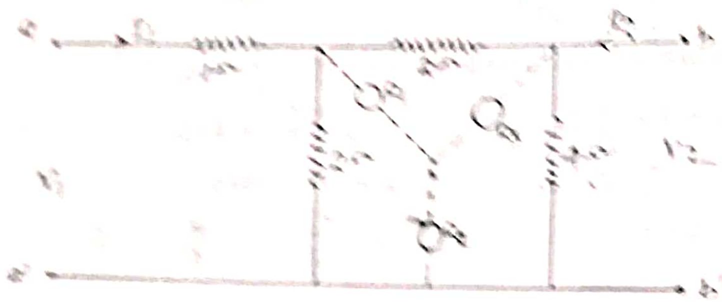
$\Rightarrow V_1 = -16I_2$

$\Rightarrow \frac{V_1}{I_2} = -16 \Rightarrow \frac{-V_1}{I_2} = 16$

(iii) $I_1 = \frac{V_1}{...}$ ... $V_2 = 0$

(iv) $I_2 = \frac{V_2}{...}$ ... $V_1 = 0$

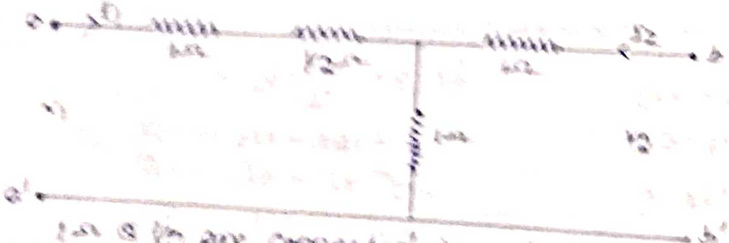
③



$R_1 = \frac{2\times2}{2+2+4} = \frac{4}{8} = \frac{1}{2}\Omega$

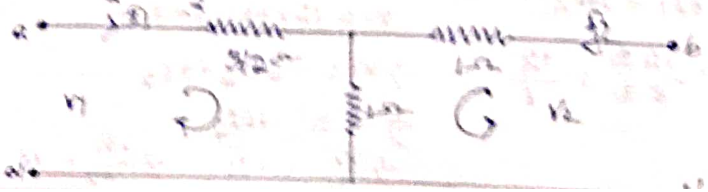
$R_2 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\Omega$

$R_3 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\Omega$



1Ω & $\frac{1}{2}\Omega$ are connected in series so

$1+\frac{1}{2} = \frac{3}{2}$



Applying KVL at loop 1:-

$V_1 = \frac{3}{2}I_1 + 1(I_1+I_2)$

$\Rightarrow V_1 = \frac{3}{2}I_1 + I_1 + I_2$

$\Rightarrow V_1 = (\frac{3}{2}+1)I_1 + I_2$

$\Rightarrow V_1 = \frac{5}{2}I_1 + I_2$ — (1)

Applying KVL at loop 2:-

$V_2 = I_2 + 1(I_1+I_2)$

$\Rightarrow V_2 = I_1 + 2I_2$ — (2)

Again

$V_1 = Z_{11}I_1 + Z_{12}I_2$ — (3)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ — (4)

Comparing eq (1) & (3) we get

$Z_{11} = 5/2 \quad Z_{12} = 1$

Comparing eq (2) & (4) we get

$Z_{21} = 1 \quad Z_{22} = 2$

$\therefore Z = \begin{bmatrix} 5/2 & 1 \\ 1 & 2 \end{bmatrix}$, $V_1 = \frac{5}{2}I_1 + I_2$, $V_2 = I_1 + 2I_2$

Conversion of "Z" to "Y":-

$Z = \begin{bmatrix} 5/2 & 1 \\ 1 & 2 \end{bmatrix}$

Cofactor $Z_{11}(5/2) = 2$

Cofactor $Z_{12}(1) = -1$

Cofactor $Z_{21}(1) = -1$

Cofactor $Z_{22}(2) = 5/2$

$\Delta Z = 2\times\frac{5}{2} - (1\times1) = 5-1 = 4$

$\therefore Z = \begin{bmatrix} 2 & -1 \\ -1 & 5/2 \end{bmatrix}$

$\therefore$ Adj of $Z = \begin{bmatrix} 2 & -1 \\ -1 & 5/2 \end{bmatrix}$

$(Z^{-1}) = \frac{\text{Adj } Z}{\Delta Z} = \begin{bmatrix} 2/4 & -1/4 \\ -1/4 & 5/8 \end{bmatrix} \Rightarrow Z^{-1} = Y = \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{bmatrix}$

Eqⁿ of Y parameters: $I_1 = \frac{1}{2}V_1 - \frac{1}{4}V_2$

$I_2 = -\frac{1}{4}V_1 + \frac{5}{8}V_2$

Conversion of Y to H :-

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

(i) $h_{11} = \frac{V_1}{I_1} | V_2 = 0$

$I_1 = \frac{1}{2}V_1 - \cancel{\frac{1}{4}V_2}^{0}$

$I_2 = -\frac{1}{4}V_1 + \cancel{\frac{5}{8}V_2}^{0}$

$\frac{V_1}{I_1} = \cancel{\quad} \frac{1}{\cancel{2}}\ 2$

(iii) $h_{21} = \frac{I_2}{I_1} | V_2 = 0$

$\Rightarrow I_1 = \frac{1}{2}V_1 - \cancel{\frac{1}{4}V_2}^{0}$

$I_2 = -\frac{1}{4}V_1 - \cancel{\frac{5}{8}V_2}^{0}$

$\frac{I_2}{I_1} = \frac{-1/4\,V_1}{1/2\,V_1} = \frac{-1}{2}$

(ii) $h_{12} = \frac{V_1}{V_2} | I_1 = 0$

$\Rightarrow I_1 = \frac{1}{2}V_1 - \frac{1}{4}V_2$

$\Rightarrow 0 = -\frac{1}{4}V_1 - \frac{1}{4}V_2$

$\Rightarrow \frac{1}{4}V_1 = -\frac{1}{4}V_2$

$\Rightarrow \frac{V_1}{V_2} = \frac{1/4}{1/2} = \frac{-1}{2}$

(iv) $h_{22} = \frac{I_2}{V_2} | I_1 = 0$

$\cancel{I_1}^{0} = \frac{1}{2}V_1 - \frac{1}{4}V_2$ — ①

$I_2 = -\frac{1}{4}V_1 - \frac{5}{8}V_2$ — ②

$\Rightarrow V_1 = \frac{1}{4}V_2 \times 2$

$V_1 = \frac{1}{2}V_2$

put the value of V_1 in eqn ②

$I_2 = -\frac{1}{4} \cdot \frac{1}{2}V_2 - \frac{5}{8}V_2$

$\Rightarrow I_2 = \left(\frac{-1}{8} + \frac{5}{8}\right)V_2$

$\Rightarrow \frac{I_2}{V_2} = \frac{-1+5}{8} = \frac{4}{8} = \frac{1}{2}$

$$H = \begin{bmatrix} 2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

eqn of H Parameter $V_1 = 2I_1 + \frac{1}{2}V_2$ — ③

$I_2 = -\frac{1}{2}I_1 + \frac{1}{2}V_2$ — ④

Conversion of Y to ABCD :-

$V_1 = AV_2 - BI_1$

$I_1 = CV_2 - DI_2$

(i) $A = \frac{V_1}{V_2} | I_2 = 0$

$I_1 = \frac{1}{2}V_1 - \frac{1}{4}V_2$

$\cancel{I_2}^{0} = -\frac{1}{4}V_1 + \frac{5}{8}V_2$

$\Rightarrow \frac{1}{4}V_1 = \frac{5}{8}V_2$

$\Rightarrow \frac{V_1}{V_2} = \frac{1/4}{5/8} = \frac{5}{4}$

(ii) $B = \frac{-V_1}{I_2} | V_2 = 0$

$I_1 = \frac{1}{2}V_1 - \cancel{\frac{1}{4}V_2}^{0}$

$I_2 = -\frac{1}{4}V_1 + \cancel{\frac{5}{8}V_2}^{0}$

$\therefore I_2 = -\frac{1}{4}V_1$

$\Rightarrow \frac{-V_1}{I_2} = 4$

(iii) $C = \frac{I_1}{V_2} | I_2 = 0$

$I_1 = \frac{1}{2}V_1 - \frac{1}{4}V_2$ - - ①

$\cancel{I_2}^{0} = -\frac{1}{4}V_1 + \frac{5}{8}V_2$ - ②

$\Rightarrow \frac{1}{4}V_1 = \frac{5}{8}V_2$

$\Rightarrow V_1 = \frac{5}{\cancel{8}_2} \cdot V_2 \times \cancel{4} = \frac{5}{2}V_2$

put the value of "V_2" in eqn ①

$\Rightarrow I_1 = \frac{1}{2} \times \frac{5}{2}V_2 - \frac{1}{4}V_2$

$\Rightarrow I_1 = V_2\left(\frac{5}{4} - \frac{1}{4}\right)$

$\Rightarrow \frac{I_1}{V_2} = \frac{5-1}{4} = \frac{4}{4} = 1$

(iv) $D = \frac{-I_1}{I_2} | V_2 = 0$

$\Rightarrow I_1 = \frac{1}{2}V_1 - \cancel{\frac{1}{4}V_2}^{0}$

$I_2 = -\frac{1}{4}V_1 + \cancel{\frac{5}{8}V_2}^{0}$

$\frac{I_1}{I_2} = \frac{1/2\,V_1}{-1/4\,V_1} = -\frac{1}{2} \times \frac{4}{1} = -2$

$\Rightarrow \frac{-I_1}{I_2} = 2$

$$\therefore ABCD = \begin{bmatrix} 5/4 & 4 \\ 1 & 2 \end{bmatrix}$$

Eqn of ABCD Parameter =

$V_1 = \frac{5}{4}V_2 - (4)I_2$

$I_1 = 1V_2 - 2I_2$

CHAPTER-04

Filters

⇒ A filter blocks unwanted signals or noise signals and passes wanted or desired signals.

⇒ A filter is basically frequency selective network that allows signals of a particular band of frequencies and rejects or attenuates signals of other frequency.

⇒ Filter networks are widely used in communication system or to separate various voice channels in carrier frequency telephone circuit.

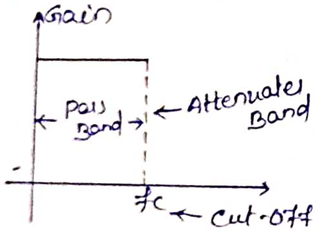
⇒ They are classified into four common types:-

(i) Low-Pass filters (LPF)
(ii) High-Pass filter (HPF)
(iii) Bandpass filter (BPF)
(iv) Band stop or Band elemination filter (BSF)

Low-Pass filter (LPF):-

⇒ An LPF passes through low frequency signal and blocks or attenuates signals that have frequences above selected cut-off frequency (f_c)

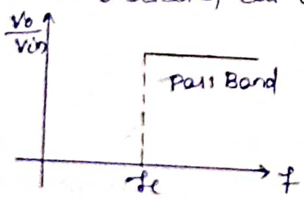
$\alpha = \frac{V_o}{V_i}$



High Pass filter (HPF):-

⇒ A HPF network will pass only those input signals whose frequencies are above the selected cut-off frequency and attenutes all frequencies below a selected cut-off frequency.

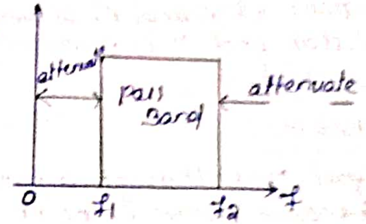
Pass Band → f_c to ∞
Stop Band → 0 to f_c



Band Pass filter (BPF):-

⇒ A BPF N/w passes frequencies betw two selected cut-off frequencies and attenuates all other frequencies.

Pass Band:- f_1 to f_2
Stop Band:- 0 to f_1
f_2 to ∞

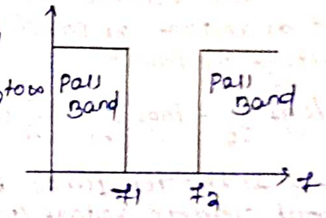


Band stop Filter:-(BSF)

⇒ A BSF N/w passes all frequencies laying outside a contain range while it attenuates all frequencies betw the two selected frequency.

⇒ It is also called Band elimination filter.

Pass Band → 0 to f_1 & f_2 to ∞
Stop Band → f_1 to f_2



Parameters of a filter:-

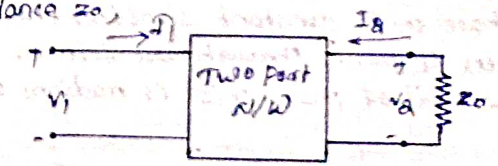
⇒ There are four important parameters that are necessary to analyse the performance of a filter network.

① Propagation Constant (γ)
② Attenuation Constant (α)
③ Phase shift Constant (β)
④ Characteristics Constant (Z_0)

① Propagation Constant (γ):-

⇒ For any two-Port network terminated by charcteristices Impedance Z_0,

$$\frac{I_1}{I_2} = \frac{V_1}{V_2} = e^{\gamma}$$



$$\gamma = \log_e \frac{I_1}{I_2} = \log_e \frac{V_1}{V_2}$$

Where, γ is known as propagation constant.

⇒ Propagation constant determines the propagation performance of any two-port Network.

$$\gamma = \alpha + j\beta$$

Where α is real part of γ and is known as attenuation constant of the filter and β is imaginary part of γ and is known as phase constant.

② **Attenuation Constant (α):-**

⇒ Whenever a signal passes through a passive filter network, it gets attenuated, because passive compount like capacitor and inductor consume some of the signal energy.

⇒ The alternation constant determains the attenution of the signal when it passes through the filter.

⇒ Attenuation can be expressed is decibals or nepers.

Neper:- It is defined as the natural log of the ratio of input current or voltage or power to the output current or voltage or power.

$$\text{Neper}(N) = \log_e \frac{I_1}{I_2} = \log_e \frac{V_1}{V_2} = \frac{1}{2}\log_e \frac{P_1}{P_2}$$

Decibel:- It is defined as the ten times the common log of the ratio of input current/voltage/power and output current/voltage/power.

$$D = 20\log_{10}\frac{I_1}{I_2} = 20\log_{10}\frac{V_1}{V_2} = 20\log_{10}\frac{P_1}{P_2}$$

⇒ **Relation Between Nepers and Decibel:-**

$$\text{Attenuation in Nepers} = \frac{\text{Attenuation in Decibels}}{8.686}$$

$$= 0.115 \times \text{Attenuation in Decibel.}$$

③ **Phase shift Constant (β):-**

⇒ When the signal passes through the filters, it gets some shift in phase.

⇒ Phase shift constant signifies the phase shift in the signal when it passes through the filters.

⇒ The unit of phase shift is radians or degrees.

④ **Characteristic Impendence (Z_0):-**

⇒ Characteristic Impendence is the image impendence of a two-port network.

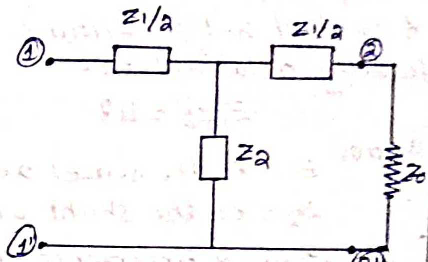
⇒ For symetrical network, the image impendence of the port 1-1 is equal to the image impendence at port 2-2'. They are equal to the charcterstics impendence Z_0.

* Inductor pass through DC (low frequency) and Block the AC (High frequency).

* Capacitor is passes through AC (High frequency) and Block the DC (low frequency).

Analysis of Filter Network:-

Symmetrical T-network:-



$$Z_{OC} = \frac{Z_1}{2} + Z_2$$

$$Z_{SC} = \frac{\frac{Z_1^2}{4} + Z_1Z_2}{Z_1/2 + Z_2}$$

① $Z_{0T} = \sqrt{Z_{OC}Z_{SC}} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$ (Characteristics Impedance)

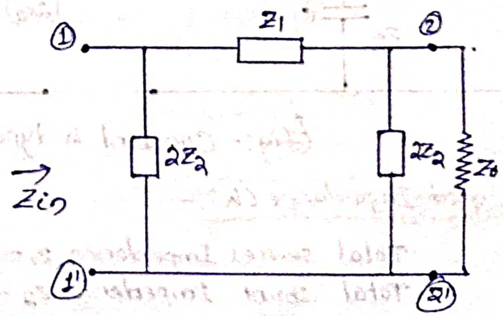
② $\gamma = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$:- (Propagation Constant)

③ $\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

④ $\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

⑤ $Z_1 + 4Z_2 = 0$ equation to obtain cutoff frequency.

Analyses of π-Network:-



① $Z_{0\pi} = \frac{Z_1Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1Z_2}}$

② $Z_{0\pi} = \sqrt{Z_{OC}\cdot Z_{SC}}$

③ $\gamma = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

④ $\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

⑤ $\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

⑥ $Z_1 + 4Z_2 = 0$

Classification of filters:-

→ Filters may be classified to be
   (i) Constant k-type or prototype filter
   (ii) m-derived filter

(i) **Constant k-type or prototype filter:-**

→ A constant k-type filter is a filter that satisfies the following relationship:-

$$Z_1 Z_2 = k^2$$

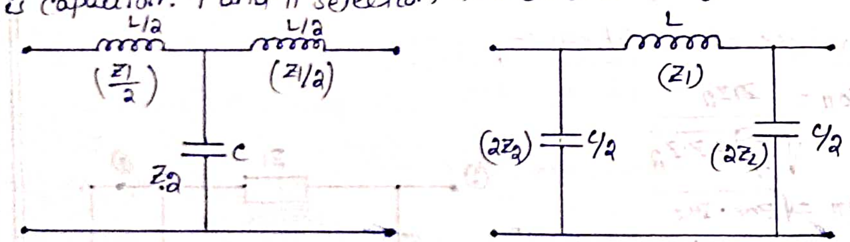
where, Z_1 = is the series arm impedence
Z_2 = is the shunt arm impedence
k = designed impedence or nominal impedence or zero characteristic impedence.

⇒ Constant k-type filter can be of T-type or π-type.

⇒ Constant k-type filters may be of low pass type and high pass type, band pass type or band-stop type.

(1) **Constant k-type low pass filter (LPF):-**

⇒ In LPF, the series element is inductor and shunt element is capacitor. T and π selection for constant k-type LPF.



(Fig:- Constant k-type filter)

Designed Impedance (k):-

Total series Impedence $Z_1 = j\omega L$
Total shunt Impedence $Z_2 = \frac{1}{j\omega C}$

For Constant k-type filters, $Z_1 Z_2 = k^2$

$$\frac{L}{C} = k^2$$

$$\Rightarrow k = \sqrt{\frac{L}{C}}$$

⇒ $K = \sqrt{\frac{L}{C}}$ This is the expression that will be used in degine of constat k LPF.

Some value of constant-k type LPF:-

(1) Design Impedence:- $k = \sqrt{\frac{L}{C}}$

(2) Design parameters:- $L = \frac{k}{\pi f_c}$ & $C = \frac{1}{k\pi f_c}$

(3) Cut-off frequency $= f_c = \frac{1}{\pi\sqrt{LC}}$

(4) Attenuation:- $\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$ in stop band

$= 0$ in pass band

(5) Phase Constant $= (\beta) = 2\sin^{-1}\left(\frac{f}{f_c}\right)$ in pass band

$= \pi$ in stop band

(6) Charcteristics Impendence:-

$$Z_{0T} = k\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

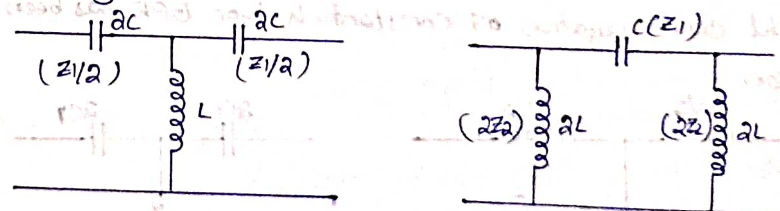
$$Z_{0\pi} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

(2) **Constant K-type High-pass filters (HPF):-**

⇒ In an HPF, the series element is a capacitor and the shunt arm element is an inductor that is.

$$Z_1 = \frac{1}{j\omega C} \text{ and } Z_2 = j\omega L$$

⇒ The ckt configuration of constant k-type HPF, both T-type and π-type are.



Some parameters value of k-type HPF:-

(1) Design Impedence $= k = \sqrt{\frac{L}{C}}$

(2) Cut-off frequency $= f_c = \frac{1}{4\pi\sqrt{LC}}$

③ Design parameters :- $L = \frac{k}{\pi f_c}$

$C = \frac{1}{4\pi f_c}$

④ Attenuation :- $\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)$ in stop Band

$\alpha = 0$ in pass band

⑤ Phase Constant :- $\beta = \pi$ in stop Band

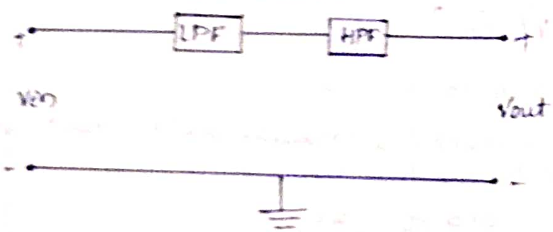
$= 2\sin^{-1}\left(\frac{f_c}{f}\right)$ in pass band

(6) Characteristics Impedance :-

$$Z_{0T} = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$Z_{0\pi} = \frac{k}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

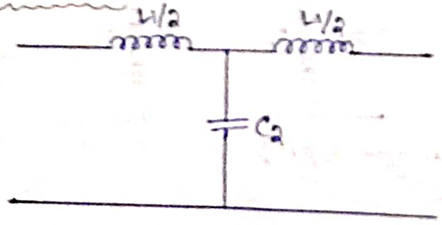
③ Constant k-type Band Pass Filter :-



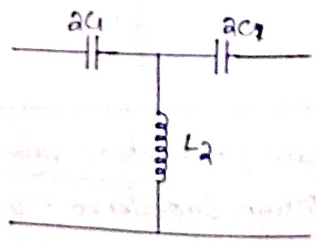
⇒ A Band pass filter can be obtained by connecting a LPF and a HPF in cascade.

⇒ The ckt configuration of constant k-type BPF has been

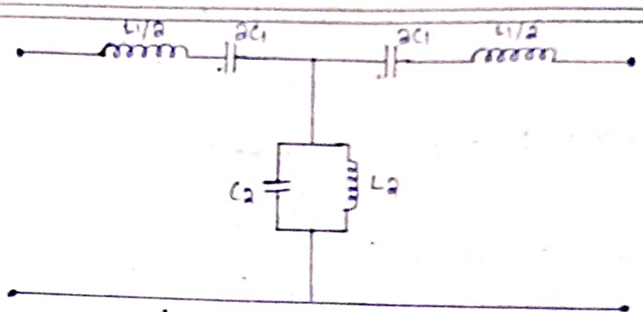
(i) T section :-



T- section of k-LPF



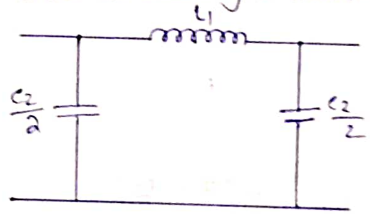
T- section of k- HPF



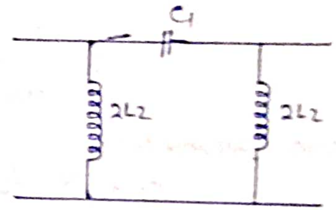
(T- section of BPF)

(ii) π Section :-

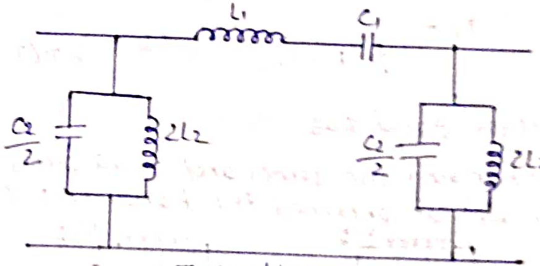
After connecting π- section is cascade of LPF & HPF are.



π-section of Constant k- LPF



π-section of HPF



(π-section of BPF).

Some parameter of constant k-type BPF :-

① Design Impedance :- $k = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}$

② Cut-off frequency :-

f_1 (upper cut off freq^n) $= \frac{k+\sqrt{k^2+\frac{L_1}{C_1}}}{2\pi L_1}$

f_2 (lower cut off freq^n) $= \frac{-\left(k+\sqrt{k^2+\frac{L_1}{C_1}}\right)}{2\pi L_1}$

③ Resonant frequency $(f_0) = \sqrt{f_1 f_2}$

(4) Attenuation $(\alpha) = 2\cosh^{-1}\sqrt{\left[\frac{\left(1-\frac{\omega^2}{\omega_0^2}\right)}{4\omega^2 L_2 C_1}\right]}$

(5) Phase Constant $(\beta) = 2\sin^{-1}\sqrt{\frac{\left(1-\frac{\omega^2}{\omega_0^2}\right)}{4\omega^2 L_2 C_1}}$

(6) Characteristics Impedence

$$Z_{0T} = k\sqrt{\frac{1-\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}{4\omega^2 L_2 C_1}}$$

$$Z_{0\pi} = \frac{k}{\sqrt{\frac{\left(1-\frac{\omega^2}{\omega_0^2}\right)}{4\omega^2 L_2 C_1}}}$$

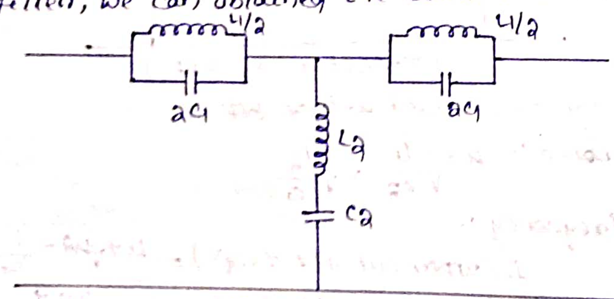
(7) Design Parameters,

$$C_1 = \frac{f_1 - f_2}{k \cdot 4\pi f_1 f_2} \; ; \; L_2 = \frac{k(f_1 - f_2)}{4\pi f_1 f_2}$$

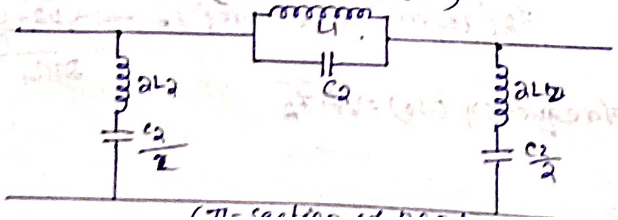
$$L_1 = \frac{k}{\pi(f_1 - f_2)} \; ; \; C_2 = \frac{1}{k \cdot \pi(f_1 - f_2)}$$

Constant k-type Band stop filter :-

⇒ By interchanging the series and shunt arms of the band pass filter, we can obtained the band-stop filter.



(T-section of BSF)



Some parameter values of constant k-type BSF :-

(1) Design Impedence :- $k = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$

(2) Cut-off frequency :-

f_1 (lower cut off freq^n) $= \frac{-1+\sqrt{1+16k^2\frac{C_1}{L_1}}}{8\pi k C_1}$

f_2 (upper cut off freq^n) $= \frac{1+\sqrt{1+16k^2\frac{C_1}{C_1}}}{8\pi C_1 k}$

(3) Attenuation (α) :-

$$\alpha = 2\cosh^{-1}\sqrt{\frac{\omega^2 L_1 C_2}{4\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}}$$

(4) Phase shift (β) :-

$$\beta = 2\sin^{-1}\sqrt{\frac{\omega^2 L_1 C_2}{4\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}}$$

(5) Resonant frequency $(f_0) = \sqrt{f_1 f_2}$

(6) Characteristics Impedence (Z_0) :-

$$Z_{0T} = \sqrt{k^2 - \frac{\omega^2 L_1^2}{4\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}}$$

$$Z_{0\pi} = \frac{k^2}{\sqrt{k^2 - \frac{\omega^2 L_1^2}{4\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}}}$$

(7) Design Parameters :-

$$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2} \; ; \; C_1 = \frac{1}{4\pi k(f_2 - f_1)}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} \; ; \; C_2 = \frac{f_2 - f_1}{4\pi f_1 f_2}$$

Example :-

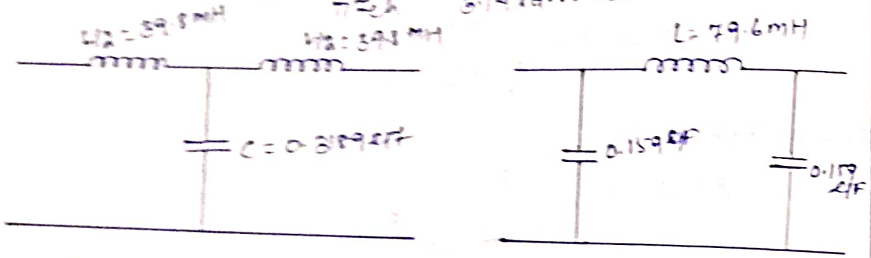
5. Design a LPF (both π & T-section) having a cut-off frequency of 2kHz to operate with a terminated load resistance of 500 Ω.

Sol^n It is given that $k = \sqrt{\frac{L}{C}} = 500\,\Omega$

$f_c = 2kHz = 2000Hz$

$L = \frac{k}{\pi f_c} = \frac{500}{3.14 \times 2000} = 79.6$ ~~mHz~~ mH

$C = \frac{1}{\pi f_c k} = \frac{1}{3.14 \times 2000 \times 500} = 0.318\ \mu F$



Example-2 :-

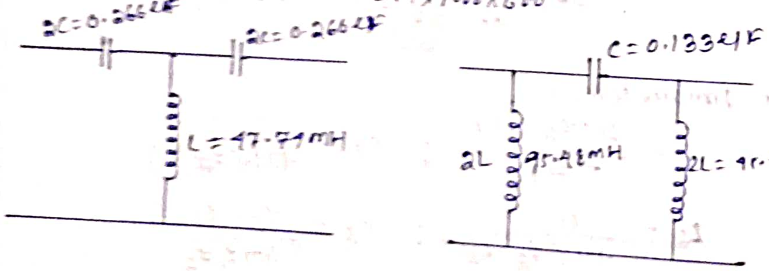
Design a HPF having a cut-off frequency of 1kHz with a load resistance of 600 Ω.

Sol^n Given

$R_L = k = 600\,\Omega$ & $f_c = 1000Hz$

$L = \frac{k}{4\pi f_c} = \frac{600}{4 \times 3.14 \times 1000} = 47.74\ mH$

$C = \frac{1}{4\pi f_c k} = \frac{1}{4 \times 3.14 \times 1000 \times 600} = 0.133\ \mu F$



Example -3 :-

Design a band-pass filter having a design impedence of 500 Ω and cut off frequencies 1kHz and 10kHz.

Solution :- Given

$k = 500\Omega$, $f_1 = 1000Hz$, $f_2 = 10000Hz$

$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{500}{\pi \times 9000} = \frac{5.55}{\pi}\ mH = 16.68\ mH.$

$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{9000}{4 \times \pi \times 500 \times 1000 \times 10000} = 0.143\ \mu F$

$L_2 = C_1 k^2 = 3.57\ mH$

$C_2 = L_1 / k^2 = 0.070\ mH$

Each of the two series arms of the constant k, T-section filter is given by

$\frac{L_1}{2} = \frac{17.68}{2} = 8.84\ mH$

$2C_1 = 2 \times 0.143 = 0.286\ \mu F$

And the shunt-arm elements of the N/w are given by.

$C_2 = 0.0707\ \mu F$ & $L_2 = 3.57\ mH$

$C_1 = 0.143\ \mu F$ & $L_1 = 16.68\ mH$

$\frac{C_2}{2} = \frac{0.707}{2} = 0.035\ \mu F$

$2L_2 = 2 \times 0.0358 = 0.0716\ mH$

Example-4

Design a band stop filter having a design impedance of 600 Ω and cut off frequencies $f_1 = 2kHz$ & $f_2 = 6kHz$

Solution

$(f_2 - f_1) = 4kHz$

$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 2000 \times 6000} = 63mH$

$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{1}{4 \times \pi \times 600(4000)} = 0.033\ \mu F$

$$L_2 = \frac{1}{4\pi(f_2 - f_1)} = \frac{1}{4\times\pi(4000)} = 12\ mH$$

$$C_2 = \frac{1}{4\pi}\left\{\frac{f_2 - f_1}{f_1 f_2}\right\} = \frac{1}{600\times\pi}\left(\frac{4000}{2000\times6000}\right) = 0.176\ \mu F$$

Each of the two series arms of the Constant = L, T-section Filter

$$\frac{L_1}{2} = \frac{63}{2} = 31.5\ mH$$

$$2C_1 = 0.033\times2 = 0.066\ \mu F$$

and shunt arm elements of the N/w are:

$$L_2 = 12mH, \quad C_2 = 0.176\ \mu F$$

→ For the constant k, π-section filter, the series arm to

$$L_1 = 63mH \quad C_1 = 0.033\ \mu F$$

Shunt arm are

$$2L_2 = 2\times12 = 24\ mH$$

$$\frac{C_2}{2} = \frac{0.176}{2} = 0.088\ \mu F.$$

Poly Phase Circuit

⇒ Poly phase system is a combination of two or more than two voltages having same magnitude and frequency but displaced form each other by an equal electrical angle.

Poly means → Many (more than one)

Phase means → Windings or circuit.

⇒ In poly phase system, the increase in the available power is not significant beyond the three-phase system that means the power generated by the same machine increases 41.4% form single phase to two phase and increase in the form single phase to two phase and increase in the power is 50% form single phase to three phase. Beyond three-phase, the maximum possible increase is only 7%, but the complications are many. So an increase beyond three phase does not justify the extra complications.

⇒ A 3-φ system of voltages (currents) is a combination of three single-phase system of voltages (currents) of which the three voltages (currents) differ in phase by 120° electrical.

Advantage of 3φ systems :-

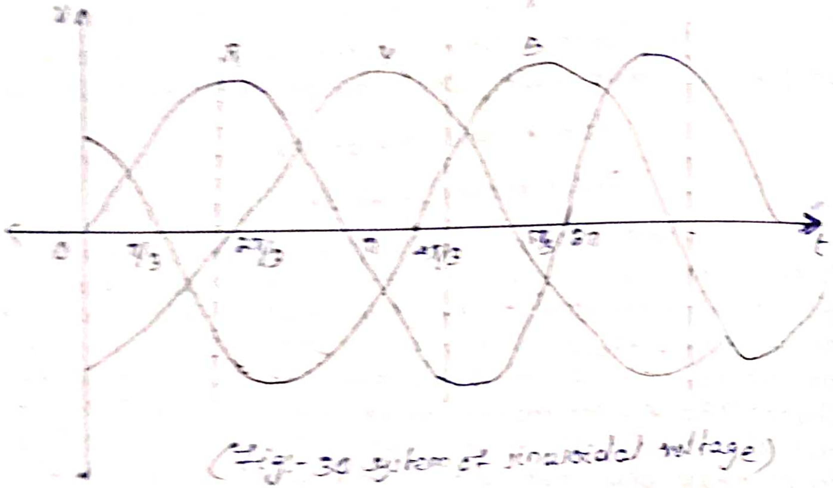
⇒ The o/p of 3φ machine generating electricity is more than o/p of a 1φ machine of same size.

⇒ The most commonly used 3φ induction motor are self starting but for 1φ motor, a separate starting winding is required.

⇒ The power factor of 3φ system is better than that of the single phase system.

⇒ Single phase supply can also be obtained form a 3φ system.

⇒ For rectification of Ac to Dc, the Dc o/p voltage becomes less fluctuating if the number of phases is increased.



(Fig:- 3φ system of sinusoidal voltage)

Phase Sequence:-

⇒ The order in which the maximum value of voltage of each phase appear is called the phase sequence.

⇒ It can be RYB or RBY

We can write in sinusoidal form

$v_{RR'} = V_m \sin\omega t$

$v_{YY'} = V_m \sin(\omega t - 120°)$

$v_{BB'} = V_m \sin(\omega t - 240°)$

Symmetrical System:- (or Balanced Supply)

⇒ In a symmetrical 3φ system, the magnitude of 3φ voltage is the same but there is a time phase difference of 120° between the voltage.

Unbalanced supply:-

⇒ A 3φ system is said to be unbalanced when either of the 3φ voltage are equal in magnitude or the phase angle between the three phase is not equal to 120°.

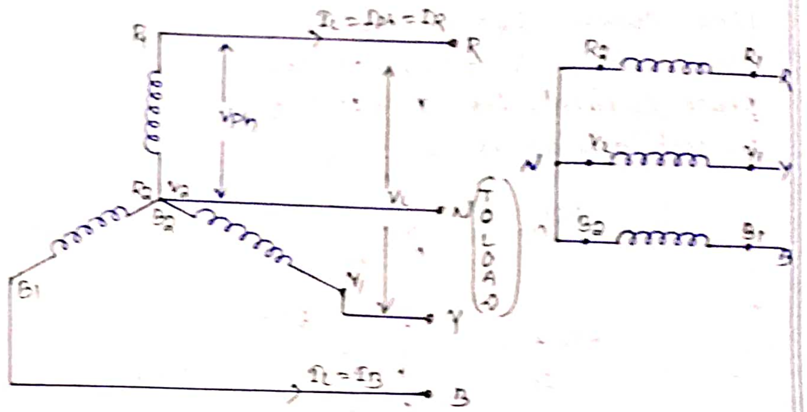
Three phase winding connection:-

⇒ A 3φ generator will have 3φ winding. Three phase winding can be connected in two ways.

1) Star Connection

2) Delta Connection.

(1) STAR CONNECTION (Y):-



⇒ The star connection formed by connecting the starting or finishing ends of all the three winding together. A fourth conductor, that is taken out of the start point is called the neutral point. The remaining three ends are brought out for connection to load.

⇒ The current flowing through each line conductor is called Line current (I_L).

⇒ In star connection, the line current is also the phase current.

⇒ The voltage across the each phase is called phase voltage (V_{Ph}) and voltage across any two line conductors is called Line voltage (V_L).

⇒ In Balanced three phase load, the sum of their current that is I_R, I_Y and I_B will be zero.

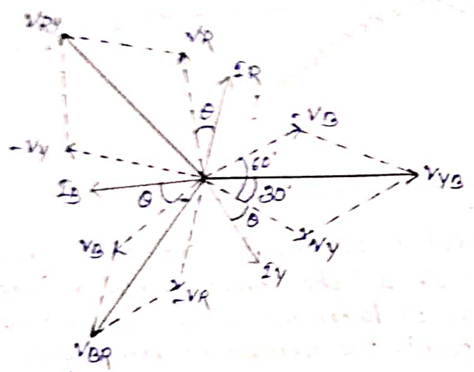
⇒ The natural wire connected between the supply neutral point and the load neutral point will carry no current for a balance system.

⇒ Considered the balanced star-connected system.

⇒ Suppose the load is inductive therefore current will be lag the applied voltage by an angle φ.

⇒ In balanced system, the magnitude of current and voltage of each phase will be the same.

Phase voltage, $V_R = V_Y = V_B = V_{ph}$
Line current, $I_R = I_Y = I_B = I_L$
Line voltage, $V_L = V_{RY} = V_{YB} = V_{BR}$
Phase current, $I_{ph} = I_R = I_Y = I_B$
In star connection, $I_L = I_{ph}$



(Fig:- Phasor Diagram)

To derive relation between V_L & V_{ph}

$V_{RY} = V_{RN} + V_{NY}$

$V_{RY} = V_{RN} + (-V_{YN})$

$V_{YB} = V_{YN} + (-V_{BN})$

$V_{BR} = V_{BN} + (-V_{RN})$

Form the phasor diagram the phase angle between phasor V_R & $(-V_Y)$ is 60°

$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60°}$

$V_{RY} = V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph} V_{ph} \times \frac{1}{2}}$

$V_L = \sqrt{3} V_{ph}$

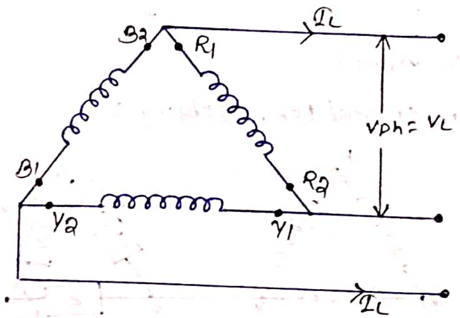
$\boxed{V_L = \sqrt{3} V_{ph}}$

Thus, for the star-connection system
→ Line voltage = $\sqrt{3}$ × phase voltage
→ Line current = phase current.

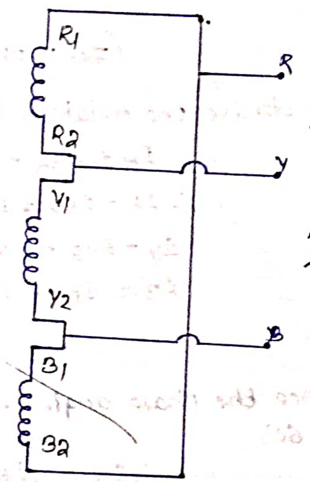
Power:-
Power output per phase = $V_{ph} I_{ph} \cos\phi$
Total power output $= 3 V_{ph} I_{ph} \cos\phi$
$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$
$P = \sqrt{3} V_L I_L \cos\phi$

∴ Power = $\sqrt{3}$ × Line voltage × Line current × P.F.

(2) Delta Connection (Δ):-



To LOAD



To LOAD

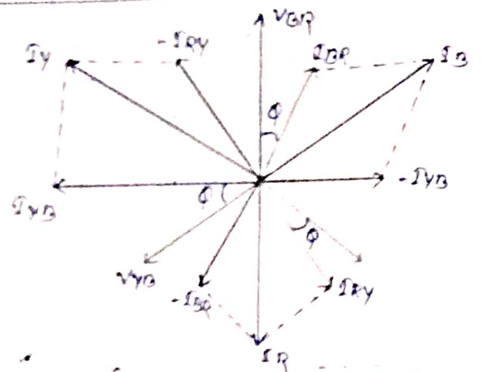
⇒ The delta connection is formed by connecting the end of one winding is starting end of the other and connections are continued to form a closed loop.

⇒ In delta connection, the current flowing through each line conductor is called line current (I_L) and the current flowing through each phase winding is called phase current (I_{ph}).

⇒ In delta connection, phase voltage is same as line voltage ($V_L = V_{ph}$)

⇒ In balanced delta connection system, the current through the phase is not the same as through the supply lines.

Line voltage ⇒ $V_L = V_{RY} = V_{YB} = V_{BR}$
Line current ⇒ $I_L = I_R = I_B = I_Y$
Phase voltage ⇒ $V_{ph} = V_{RY} = V_{YB} = V_{BR}$
Phase current ⇒ $I_{ph} = I_{RY} = I_{YB} = I_{BR}$



(Fig: Phasor Diagram)

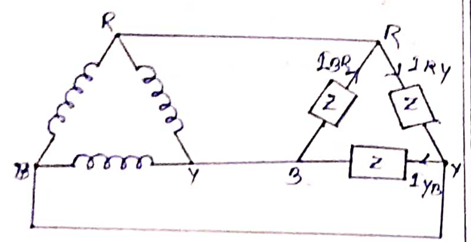
⇒ To derive the relation between I_L and I_{ph}, applying KCL.

$I_R + I_{BR} = I_{RY}$

$\therefore I_R - I_{RY} = I_{BR}$

$I_Y - I_{YB} - I_{RY}$

$I_B = I_{BR} - I_{YB}$



Since the phase angle between phase current & I_{RY} and $-I_{BR}$ is 60°.

$\therefore I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2 I_{RY} I_{BR} \cos 60^\circ}$

$I_R = I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} \cdot I_{ph} \times \frac{1}{2}}$

$I_R = \sqrt{2 I_{ph}^2 + I_{ph}^2}$

$\boxed{I_R = \sqrt{3} I_{ph}}$

⇒ For delta connected 3-φ system.

→ Line current = √3 × phase current

→ Line voltage = phase voltage.

Power:-

Power o/p per phase = $V_{ph} I_{ph} \cos\phi$

Total power output = $3 V_{ph} I_{ph} \cos\phi$

$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$

$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$

Power = √3 × Line voltage × Line current × P.F

⇒ If per phase power P_h and total power is P_T, then

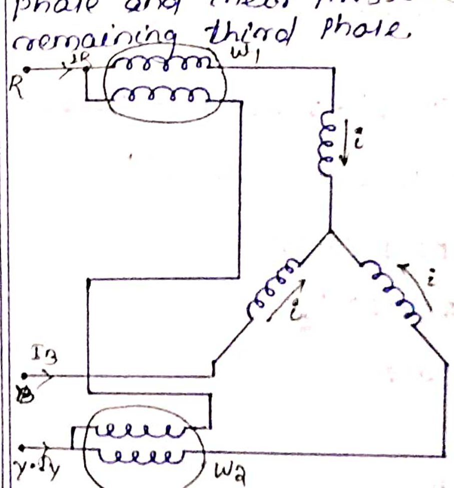
$\boxed{P_T = 3P_h}$

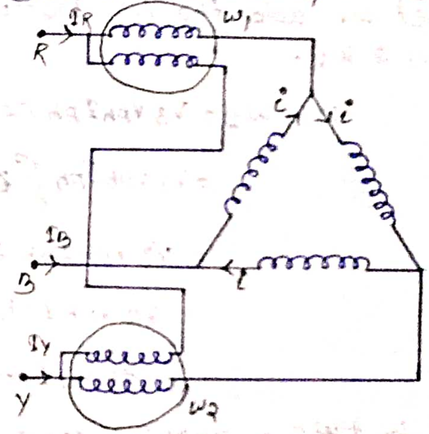
Measurement of Power in Three-Phase Circuit:-

Two wattmeter method:-

⇒ This method requires only two wattmeters to measure three-phase load for balanced as well as unbalanced loads.

⇒ In this method, two wattmeters are connected in two phase and their pressure coils are connected to the remaining third phase.

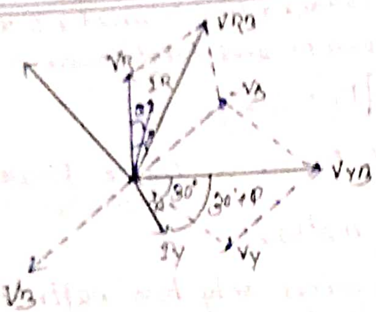




⇒ Consider the measurement of 3φ system of Y-connected load.

→ W_1, W_2 be the two wattmeter reading.

⇒ The current flowing through the current coil of wattmeter W_1 is I_R. The voltage appearing across it pressure coil is V_{RB}. The wattmeter reading W_2 will be equal to $W_1 = V_{RB} I_R \cos$ of angle betw V_{RB} and I_R. Similarly the wattmeter reading W_2 will be equal to $W_2 = V_{YB} I_B \cos$ of angle between V_{YB} and I_B.



$W_1 = V_{RB} I_L \cos(30-\phi) = \sqrt{3} V_{ph} I_{ph} \cos(30-\phi) = V_L I_L \cos(30-\phi)$

$W_2 = V_{YB} I_Y \cos(30+\phi) = \sqrt{3} V_{ph} I_{ph} \cos(30+\phi) = V_L I_L \cos(30+\phi)$

Let we add the two wattmeter readings that is W_1 & W_2.

$$W_1 + W_2 = \sqrt{3} V_{ph} I_{ph} \cos(30-\phi) + \sqrt{3} V_{ph} I_{ph} \cos(30+\phi)$$

$$= \sqrt{3} V_{ph} I_{ph} \{\cos(30-\phi) + \cos(30+\phi)\}$$

$$= \sqrt{3} V_{ph} I_{ph} \, 2 \cos\phi \cos 30°$$

$$= \sqrt{3} V_{ph} I_{ph} \, 2 \cos\phi \times \frac{\sqrt{3}}{2}$$

$$= 3 V_{ph} I_{ph} \cos\phi$$

$W_1 + W_2 = 3 V_{ph} I_{ph} \cos\phi$

or $\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi}$ — (i)

Thus it is proved that the sum of the wattmeter reading is equal to the 3-ϕ power.

⇒ Now when the two wattmeter readings are subtracted from each other, we obtained.

$\Rightarrow W_1 - W_2 = \sqrt{3} V_{ph} I_{ph} \{\cos(30-\phi) - \cos(30+\phi)\}$

$W_1 - W_2 = \sqrt{3} V_{ph} I_{ph} \, 2 \sin\phi \cdot \sin 30°$

$\Rightarrow \sqrt{3}(W_1 - W_2) = \sqrt{3} \times \sqrt{3} V_{ph} \cdot I_{ph} \cdot 2 \sin\phi \cdot \sin 30°$

$\Rightarrow \boxed{\sqrt{3}(W_1 - W_2) = \sqrt{3} V_L I_L \sin\phi}$ — (ii)

Dividing eqn (ii) by (i)

$$\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\Rightarrow \phi = \tan^{-1} \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\Rightarrow \cos\phi = \cos\left[\tan^{-1}\left\{\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right\}\right] \rightarrow \text{Power factor}$$

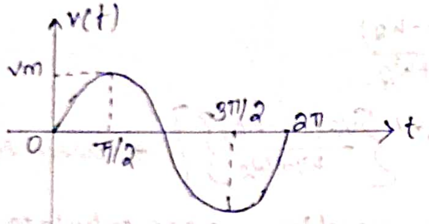
⇒ From two wattmeter reading we can calculate the total active and Reactive power and the P.F of the ckt.

CHAPTER-5

AC Circuit

The sin wave is represented by the equation.

$v(t) = V_m \sin\omega t$.



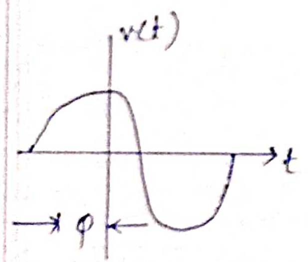
(Reference sine wave form)

(i) If a sinwave is shifted to the left of the reference wave by a certain angle ϕ.

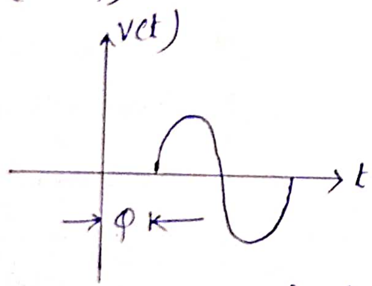
$v(t) = V_m \sin(\omega t + \phi)$

(ii) If a sine wave is shifted to the right of the reference wave by a certain angle ϕ.

$v(t) = V_m \sin(\omega t - \phi)$



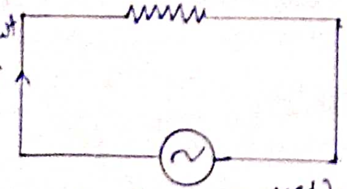
(lead waveform)



(lagging waveform)

① A.C through pure resistor :-

$i(t) = I_m \sin\omega t = I_m\angle 0^\circ$



$v(t) = V_m \sin\omega t = V_m\angle 0^\circ$

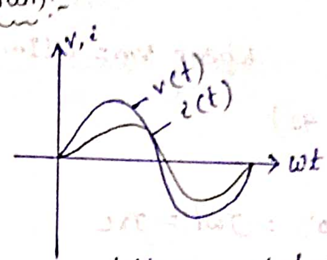
$\Rightarrow v(t) = i(t)R$

$\Rightarrow V_m \sin\omega t = I_m R \sin\omega t$

$\Rightarrow \boxed{V_m = I_m R}$

or $V_m\angle 0 = I_m\angle 0\, R$

⇒ Phasor diagram :-



⇒ There is no phase difference between these two waveform.

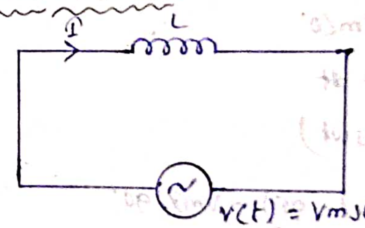
Impedence :-

In case of resistor

$$Z = \frac{V_m\angle 0^\circ}{I_m\angle 0} = R$$

$\boxed{Z = R}$

② Ac through pure Inductor :-



$v(t) = V_m \sin\omega t$

$\Rightarrow V_L = L\frac{di}{dt}$

$\Rightarrow i = i(t) = I_m \sin\omega t = I_m\angle 0^\circ$

$v(t) = L\frac{d}{dt}(I_m \sin\omega t)$

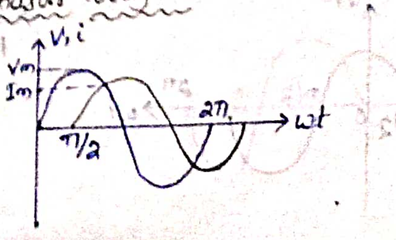
$v(t) = L\omega I_m \cos\omega t$

$v(t) = \omega L I_m \cos\omega t$

$v(t) = V_m \cos\omega t$ or $V_m \sin(\omega t + 90^\circ)$ or $V_m\angle 90^\circ$

where, $V_m = \omega L I_m = X_L I_m$

Phasor diagram



⇒ In a pure inductor the voltage and current are out of phase. The current lages behind the voltage by 90°.

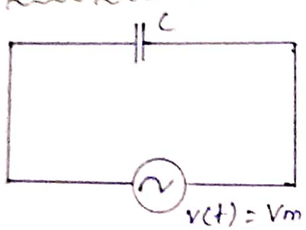
Impedence:-

$$Z = \frac{V_m \sin(\omega t + 90)}{I_m \sin \omega t} \qquad \text{where } V_m = \omega L I_m$$

$$Z = \frac{I_m \omega L \sin(\omega t + 90^\circ)}{I_m \sin \omega t}$$

$$Z = \omega L\, I_m \angle 90^\circ / I_m \angle 0^\circ = J\omega l = JX_L$$

(3) **A.C through in a pure capacitor:-**



$v(t) = V_m \sin \omega t$

$\Rightarrow v(t) = \frac{1}{C}\int i(t)\, dt$

$\Rightarrow i(t) = I_m \sin \omega t = I_m \angle 0^\circ$

$\Rightarrow v(t) = \frac{1}{C}\int I_m \sin \omega t\, dt$

$\Rightarrow v(t) = \frac{1}{\omega C} I_m(-\cos \omega t)$

$$v(t) = \frac{I_m}{\omega C}\sin(\omega t - 90^\circ) = V_m \angle -90^\circ$$

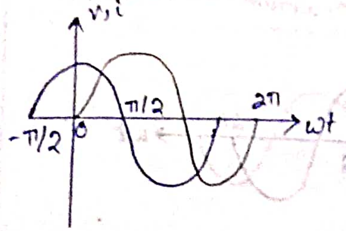
where, $V_m = \frac{I_m}{\omega C}$

$$Z = \frac{V_m \angle -90^\circ}{I_m \angle 0^\circ} = \frac{-j}{\omega C}$$

$$Z = \frac{-J}{\omega C} = -jX_C$$

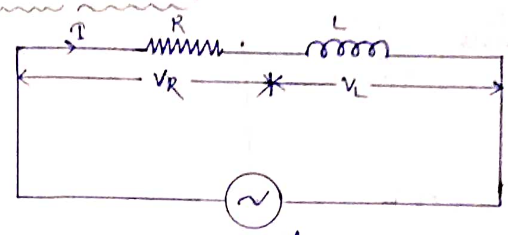
$X_C = \frac{1}{\omega C}$ = is called capacitive reactance.

Phasor diagram:-

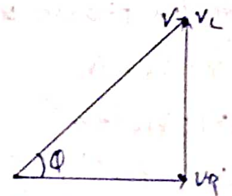


→ In a pure capacitor, the current leads the voltage by 90°.

(1) **R-L Series Circuit:-**



$$\boxed{V = V_R + jV_L}$$



· Complex phasor voltage

In magnitude, $|V| = \sqrt{V_R^2 + V_L^2}$

In Angle, $\angle V = \tan^{-1}\left(\frac{V_L}{V_R}\right)$

form complex phasor voltage

$\Rightarrow V = V_R + jV_L$

$\Rightarrow V = IR + jIX_L$

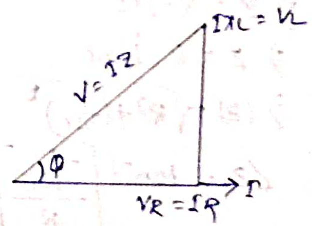
$\Rightarrow V = I(R + j\omega L)$

$\Rightarrow \frac{V}{I} = R + j\omega l$

$\Rightarrow Z = R + j\omega l$

$\Rightarrow |Z| = \sqrt{R^2 + (\omega l)^2}$

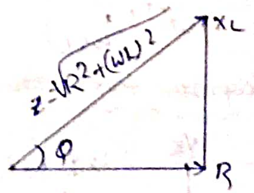
$\Rightarrow \angle Z = \tan^{-1}\left(\frac{\omega l}{R}\right)$



Power factor:-

$\cos\phi = \frac{V_R}{V}$

Impedence triangle for R-L cut:-



$\Rightarrow \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$

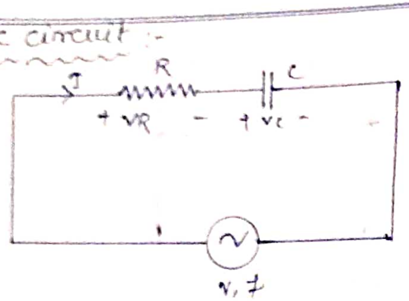
$Z\cos\phi = R$

$\Rightarrow \sin\phi = \frac{X_L}{Z}$

$\Rightarrow \tan\phi = \frac{X_L}{R}$

$\Rightarrow \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

(2) Series R-C circuit :-



Complex phasor voltage v is given by,

$\Rightarrow V = V_R - jV_C$

$|V| = \sqrt{V_R^2 + V_C^2}$

$\angle V = \tan^{-1}\left[\frac{-V_C}{V_R}\right]$

form complex phasor voltage,

$\Rightarrow V = V_R - jV_C$

$\Rightarrow V = IR - jIX_C$

$\Rightarrow \frac{V}{I} = R - jX_C$

$\Rightarrow Z = R - jX_C$

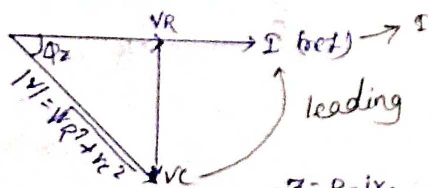
$\Rightarrow Z = R - j/\omega C \qquad \left(X_C = \frac{1}{\omega C}\right)$

$\Rightarrow |Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\angle Z = \tan^{-1}\left[\frac{-\frac{1}{\omega C}}{R}\right]$

$\angle Z = -\tan^{-1}\left(\frac{1}{\omega RC}\right)$

Phasor diagram for series RC ckt :-



$\rightarrow$ I leads by ϕ_Z from V

$Z = R - jX_C$

$\phi_Z = -V_C$

$\phi_Z = \frac{\phi_V}{\phi_I}$

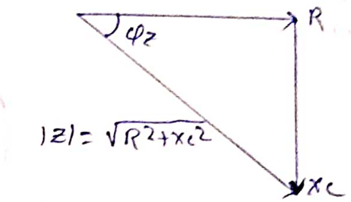
$\phi_Z = \phi_V - \phi_I$

$\phi_Z \angle 0°$

$\phi_V - \phi_I \angle 0$

$\phi_I > \phi_V$ (leading p.f.)

Impedence Triangle :-



Power factor :-

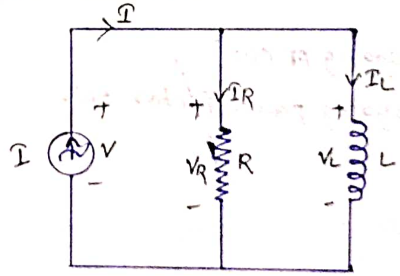
$\cos\phi = \frac{V_R}{V}$

$\cos\phi = \frac{IR}{IZ}$

$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{R}{\sqrt{R^2 + X_C^2}}$

$\Rightarrow$ Series RC N/W is referred as leading p.f. ckt.

case-4 Parallel RL Network :-



Complex phasor current is i.e. given by

$I = I_R - jI_L$

$\Rightarrow I = I_R - jI_L$

$|I| = \sqrt{I_R^2 + I_L^2}$

$\angle I = \tan^{-1}\left[\frac{-I_L}{I_R}\right]$

$\boxed{\angle I = -\tan^{-1}\left[\frac{I_L}{I_R}\right]}$

$\boxed{\angle I = -\tan^{-1}\left[\frac{x}{R}\right]}$

$\boxed{\phi_I = -\tan^{-1}\frac{I_L}{I_R} = -\tan^{-1}\frac{x}{R}}$

$\Rightarrow V_R = V_L = V$ (Red. voltage)

$\Rightarrow I = \frac{V}{R} - j\frac{V}{X_L}$

$\Rightarrow \frac{I}{V} = \frac{I}{R} - \frac{j}{X_L}$

$\Rightarrow Y = G - jB \quad \left[G = \frac{1}{R}, B = \frac{1}{X_L}\right]$

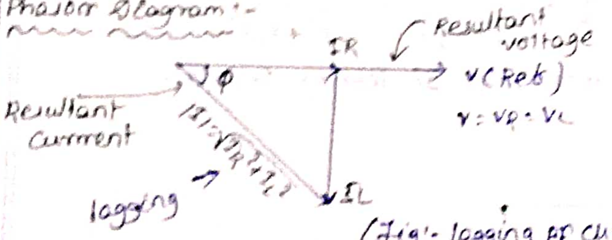
$|Y| = \frac{1}{R} - j/X_L$

$|Y| = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$

$\angle Y = \phi_Y = \tan^{-1}\left[\frac{-1/X_L}{1/R}\right]$

$\boxed{\phi_Y = -\tan^{-1}\left[\frac{R}{X_L}\right]}$

Phasor Diagram :-



(Fig:- lagging PF cut)

$\Rightarrow$ Parallel RL ckt is referred also lagging power factor ckt.

Power factor:-

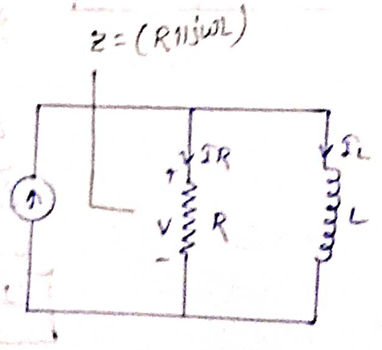
$\cos\phi = \frac{I_R}{I} = P.F$

$\cos\phi = \frac{V/R}{V/Z} = \frac{Z}{R}$

$Z = \frac{1}{Y}$

$\boxed{\cos\phi = \frac{1}{RY}}$

$Z = (R \| j\omega L)$



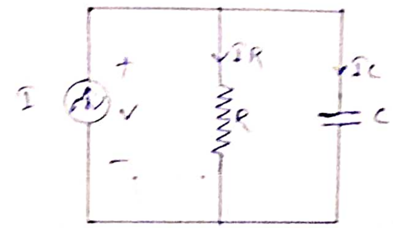
$Z = \frac{R \times j\omega L}{R + j\omega L}$

$Z = \frac{j\omega R L}{R + j\omega L}$

$\cos\phi = \left|\frac{Z}{R}\right| = \left(\frac{R\omega L}{\sqrt{R^2 + \omega^2 L^2}}\right) \times \frac{1}{R}$

$\boxed{\cos\phi = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}} = \frac{1}{RY}$

Parallel RC circuit :-

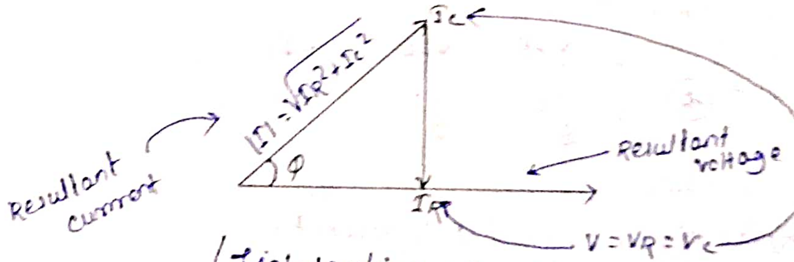


Complex phasor current I is given by

$I = I_R + jI_C$ ---- ①

$V = V_R = V_C$

$|I| = \sqrt{I_R^2 + I_C^2}$



(Fig:- leading P.F ckt)

$\Rightarrow$ Parallel RC network is referred as leading P.F ckt like series RC ckt.

$\Rightarrow I = I_R + jI_C$

$\Rightarrow I = \frac{V}{R} + j\frac{V}{X_C}$

$\Rightarrow \frac{I}{V} = \frac{1}{R} + j/X_C \quad \left[X_C = \frac{1}{\omega C}\right]$

$\Rightarrow Y = \frac{1}{R} + j/\frac{1}{\omega C}$

$\Rightarrow \boxed{|Y| = \sqrt{\frac{1}{R^2} + \omega^2 C^2}}$

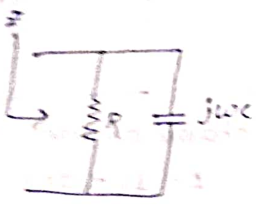
$\Rightarrow \boxed{\angle Y = \phi_Y = \tan^{-1}[R\omega C]}$

$\phi_Y = +ve$
P.F = leading.

or $\cos\phi = \frac{I_R}{I} = \frac{V/R}{V/Z} = \frac{Z}{R} = \frac{1}{RY}$

$\cos\phi = \frac{1/R}{\sqrt{\frac{1}{R^2} + \omega^2C^2}}$

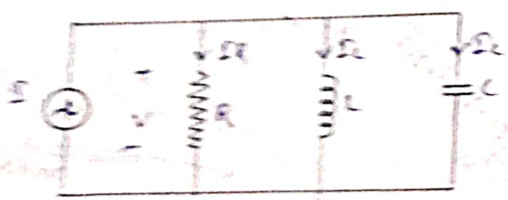
$$\boxed{\cos\phi = \frac{1}{\sqrt{1+R^2\omega^2C^2}}}$$



(b) $Z = \frac{R \times 1/j\omega C}{R + 1/j\omega C} = \frac{R}{1+Rj\omega C}$

$$\boxed{\cos\phi = \left|\frac{Z}{R}\right| = \frac{1}{\sqrt{1+R^2\omega^2C^2}}}$$

Parallel RLC circuit

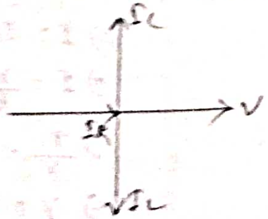


Complex phasor current I is given by,

$I = I_R + jI_C - jI_L$ $\quad [V_R = V_L = V_C = V]$

$I = I_R + j[I_C - I_L]$

$$\boxed{|I| = \sqrt{I_R^2 + (I_C - I_L)^2}}$$



$|I| = \sqrt{I_R^2 + I_C^2 + I_L^2}$

$I = I_R + j(I_C - I_L)$

$I = \frac{V}{R} + j\left(\frac{V}{X_C} - \frac{V}{X_L}\right)$

$\frac{I}{V} = Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$

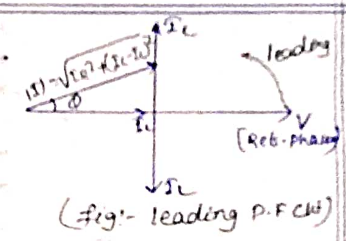
Case - 6.1 :-

$I_C > I_L$

$I = I_R + j(I_C - I_L)$

$I = I_R + jI_X$

$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$



(fig:- leading P.F ckt)

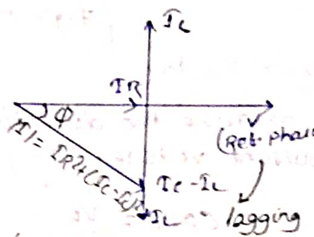
Case - 6.2

$I_L > I_C$

$I = I_R + j(I_C - I_L)$

$I = I_R - jI_X$

$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$



(fig:- lagging P.F ckt)

Case - 6.3 :-

$I_L = I_C$

$I = I_R + j(I_C - I_L)$

$I = I_R + j0$

P.F $= \cos 0° = 1$ [UPF]

$\cos\phi = \frac{1}{YR} = \frac{1/R}{Y} = \frac{1/R}{1/R} = 1$ [UPF]

$Y = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \sqrt{\frac{1}{R^2} + 0} = \frac{1}{R}$

$\cos\phi = 1$

$I_L = I_C$

$\frac{V}{X_L} = \frac{V}{X_C}$

$\frac{1}{X_L} = \frac{1}{X_C}$

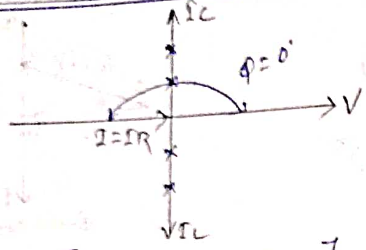
$\Rightarrow X_L = X_C$

$\Rightarrow \omega L = \frac{1}{\omega C}$

$\Rightarrow \omega^2 = \frac{1}{LC}$

$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$ $\quad \Rightarrow \boxed{\cos\phi = \frac{Z}{R} = \frac{R}{R} = 1}$ $\quad \boxed{Y = \frac{1}{R} \text{ so } Z = R}$

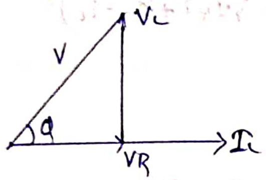
$\Rightarrow f_0 = \frac{1}{\ldots}$ Hz



[Fig:- phasor form]

Power factor :-

⇒ It describe the direction of resultant current w.r.t resultant voltage. $\left(\frac{I}{V}\right)$



$$\cos\phi = \frac{V_R}{V}$$

$$\cos\phi = \frac{IR}{V}$$

$$\cos\phi = \left(\frac{I}{V}\right)R$$

⇒ In case of leading P.F, resultant current will lead form resultant voltage.

⇒ In case of lagging P.F, resultant current will lag form resultant voltage.

Power and power triangle :-

Power = $VI\cos\phi$ = Active power

$Q = VI\sin\phi$ = Reactive power

Apparent power(S) = rms value of voltage × rms value of current

$$S = V \times I$$

⇒ The apparent power is expressed in volt ampere, that is VA or in kilo-volt ampere is kVA.

Then Real or active power = $VI \times \cos\phi$

P = Apparent power × power factor (watt or killowatt)

Reactive power $Q = VI\sin\phi$ (VAR or kilo VAR)

Q = apparent power × $\sin\phi$

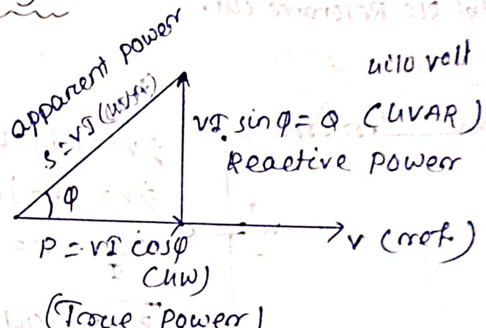
So, these 3 types of powers are related as given in the following :-

$$S^2 = P^2 + Q^2$$

$$S = \sqrt{P^2 + Q^2}$$

$$kVA = \sqrt{(kW)^2 + (kVAR)^2}$$

Power Traingle :-



Active power :-

The power which is actually consumed or utilised in an A.C circuit is called True power or Active power or real power.

⇒ It is measured in kW or MW.

Reactive power :-

The power which flow back and fourth that means it moves in both directions in the circuit or reacts upon itself is called reactive power.

⇒ It is measured in kVAR or MVAR.

RESONANCE

⇒ Resonance describe the network condition in which :-

(i) Inductive and capacitive effect neutralised to each other that means $X_L = X_C$

(ii) Power factor of the ckt will be unity.

(iii) Impedence or Admittance of the ckt will become resistive.

(iv) Input voltage and input current will be inphase of ckt that means angle between input voltage & input current will be zero.

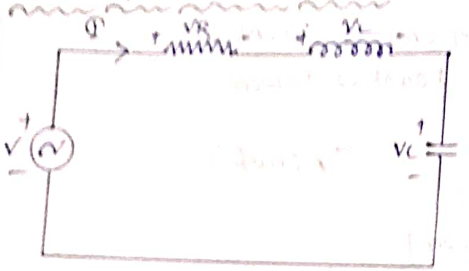
⇒ Resonance describes the energy transformation between inductor and capacitor, the frequency at which energy transferring is referred as Resonance frequency or Oscillation frequency.

⇒ Example of Resonance:-

① Series RLC Resonance ckt.

② Parallel RLC Resonance ckt.

Series RLC Resonance ckt:-



$\Rightarrow V = V_R + j(V_L - V_C)$

$\Rightarrow V = IR + j(IX_L - IX_C)$

$\Rightarrow \frac{V}{I} = Z = R + j(\omega L - \frac{1}{\omega C})$

$\Rightarrow \boxed{Z = R + j(\omega L - \frac{1}{\omega C})}$

Condition of Resonance:-

(i) $Im[Z] = 0$

or

$Im[V] = 0$

or

$Im[Y] = 0$

$\Rightarrow V_L - V_C = 0$

$\Rightarrow V_L = V_C$

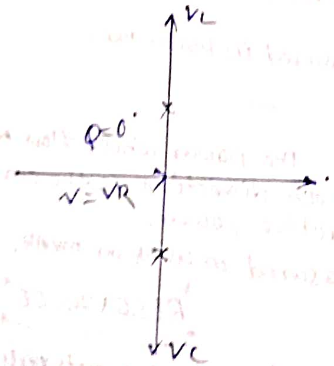
$\Rightarrow IX_L = IX_C$

$\Rightarrow X_L = X_C$

$\omega_0 L = \frac{1}{\omega_0 C}$

$\omega_0^2 = \frac{1}{LC}$

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$ → Resonant frequency



(Phasor diagram)

or

$Z = R + j(\omega L - \frac{1}{\omega C}) = 0$

$\Rightarrow \omega L - \frac{1}{\omega C} = 0$

$\Rightarrow \omega_0 L = \frac{1}{\omega C}$

$\Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ [Resonant frequency for series RLC]

Power factor:-

Input P.F = supply P.F = P.F = $\cos\phi$

$\cos 0^\circ = 1$ [V and I angle is 0]

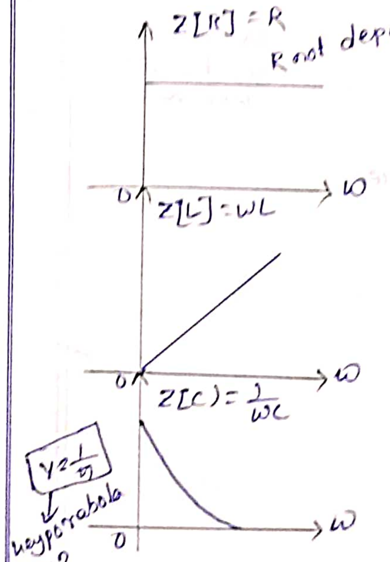
∴ So, at resonant ckt P.F will be unity.

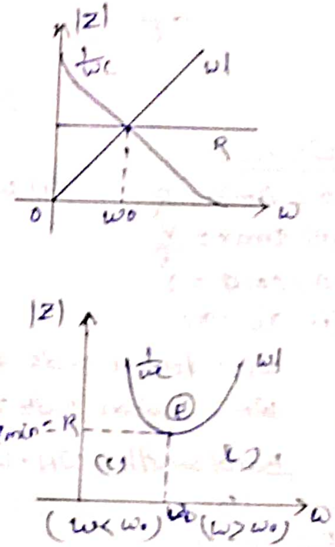
$Z = R + j(\omega L - \frac{1}{\omega C})$

$|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$;

$\angle Z = \phi_Z = \tan^{-1}\left[\frac{\omega L - \frac{1}{\omega C}}{R}\right]$

Diagram Representation:-



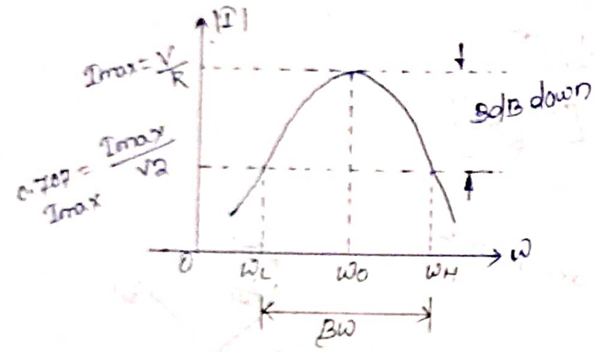


ω	R/L/C	PF
ω<ω0	Capacitive (C)	leading P.F
ω=ω0	Resistive (R)	Unit P.F
ω>0	Inductive (L)	lagging P.F

$\Rightarrow |I| = \frac{V}{|Z|}$

$|I| = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

$|I_{max}|_{\omega = \omega_0} = \frac{V}{Z_{min}} = \frac{V}{R}$



$\left(\frac{1}{\sqrt{2}} \xrightarrow{dB} 20\log\frac{1}{\sqrt{2}} = -3.0103\right.$

for '−' sign so, it is down wore value)

$\Rightarrow$ $\omega = \omega_0$:-

(i) $Z_{min} = R$ (v) $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec

(ii) $I_{max} = \frac{V}{R}$

(iii) $\cos\phi = 1$

(iv) $X_L = X_C$

ω_L = lower 3-dB frequency

ω_H = higher 3-dB frequency

Band width $= \omega_H - \omega_L$ (rad/sec)
(BW)

Bandwidth (B.W) :-

$B.W = \omega_H - \omega_L$

$BW = \frac{R}{2L} - \left(-\frac{R}{2L}\right)$

$\boxed{BW = \frac{R}{L}}$ rad/sec

$\boxed{BW = \frac{R}{2\pi L}\ Hz} : \frac{rad}{sec \times rad} = \frac{1}{sec} = Hz$

$BW = f(R, L)$

$BW \neq f(C)$

$\Rightarrow$ Quality factor :- (Q. factor)

$\Rightarrow$ Quality factor describes the energy storage capability of Inductor and capacitor in RLC network.

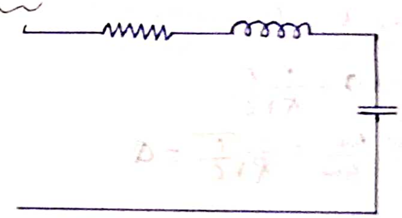
$\Rightarrow$ High value of quality factor represent high energy storage capability of network.

$Q[L] = 2\pi \times \frac{\text{Energy stored by inductor}}{\text{Energy displaced by resistor per cycle}}$

$Q[C] = 2\pi \times \frac{\text{Energy stored by capacitor}}{\text{Energy displaced by resistor per cycle}}$

Resonance :-

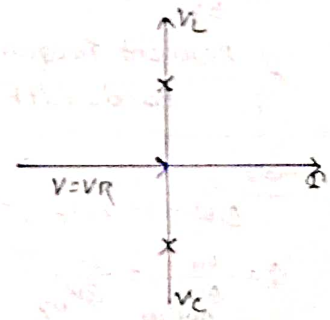
(1) Series RLC



at resonance condition

when, $\omega = \omega_0$

$V_R = V$



For Inductor

$\Rightarrow Q[L] = \frac{V_L}{V_R} = \frac{V_L}{V}$

$\Rightarrow Q[L] = \frac{\omega_0 L}{R} = \frac{\omega L}{R}$

where. $\omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow Q[L] = \frac{L}{R} \times \omega_0 = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\Rightarrow Q[L] = \frac{1}{R}\sqrt{\frac{L}{C}}$ (for resonance condition)

For Capacitor :-

$\Rightarrow Q[C] = \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{V_C}{V_R}$

at $\omega = \omega_0$

$V_R = V$

$Q[C] = \frac{V_C}{V} = \frac{V_C}{V_R}$

$Q[C] = \frac{1}{R\omega C} = \frac{1}{R\omega_0 C}$

$Q[C] = \frac{1}{R\omega_0 C} = \frac{1}{RC} \times \frac{1}{\omega_0} = \frac{1}{RC} \times \frac{\sqrt{LC}}{1}$

$\boxed{Q[C] = \frac{1}{R}\sqrt{\frac{R}{C}}}$ (for resonance condition)

⇒ Quality factor of series RLC

$\boxed{\begin{array}{l} Q_{series} = \frac{1}{R}\sqrt{\frac{L}{C}} \\ Q_{series} = f(R, L, C) \end{array}}$

Basic Result of series RLC Resonance ckt :-

(i) $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec (iii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

(ii) $BW = \frac{R}{L}$ rad/sec (iv) $\frac{\omega_0}{BW} = \frac{1}{R}\sqrt{\frac{L}{C}} = Q$

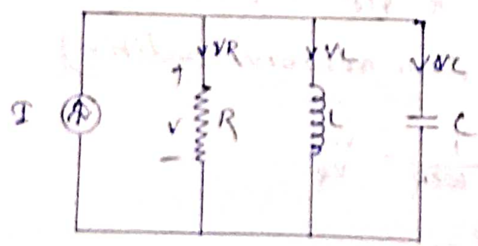
$\boxed{\begin{array}{l} \Rightarrow \frac{\omega}{BW} = Q \\ \Rightarrow \frac{\text{Resonant frequency}}{\text{Bandwidth}} = \text{Quality factor} \end{array}}$

Example

$BW = 10\,MHz$, $f_0 = 1\,MHz$

$Q = \frac{\omega_0}{BW_{rad/sec}} = \frac{f_0}{BW_{Hz}} = \frac{1 \times 10^6}{10 \times 10^6} = 0.1$

② Parallel RLC Resonance Circuit :-



$\Rightarrow I = I_R + j(I_C - I_L)$ ...... ①

$\frac{I}{V} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$

$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$ ...... ②

Condition of Resonance

$Im[I] = 0$

$Im[Y] = 0$

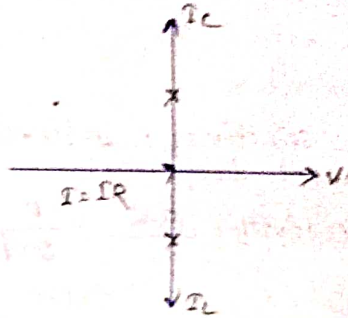
$\Rightarrow I_C - I_L = 0$

$\Rightarrow I_C = I_L$

$\Rightarrow \frac{V}{X_C} = \frac{V}{X_L}$

$\Rightarrow X_L = X_C$

$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$



$\left\{ \begin{array}{l} \text{Fig:- } \phi = 0 \\ \cos\phi = 1 \\ \text{UPF} \end{array} \right\}$

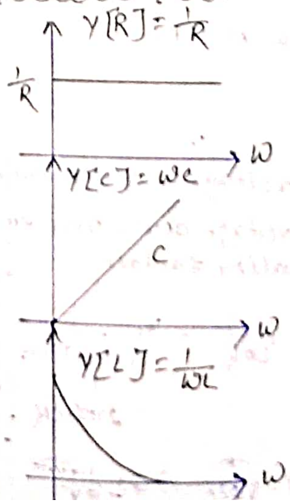
(or) $Im[Y] = 0$

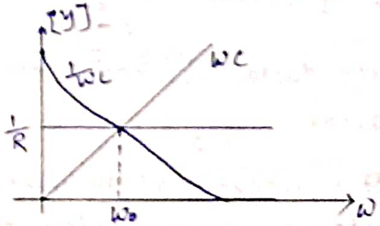
$\omega_0 C - \frac{1}{\omega_0 L} = 0$

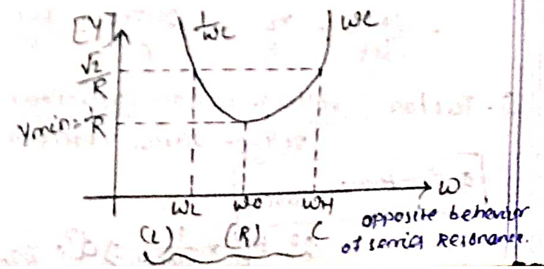
$\omega_0^2 = \frac{1}{LC}$

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$

Diagram Representation :-







opposite behavior of series Resonance.

ω	R/L/C	P.F
ω<ω₀	L	lagging P.F
ω=ω₀	R	UPF
ω>ω₀	C	leading P.F

Selectivity →

Selectivity is defined as the ratio of Band width to resonant frequency.

$$\text{selectivity} = \frac{B.W}{f_0} = \frac{R}{2\pi f} \qquad \text{selectivity} = \frac{R}{2\pi f L}$$

Quality factor (Q factor) :-

It is defined as the ratio of $2\pi \times$ maximum energy stored to energy dissipated per cycle.

$$Q\text{ factor} = \frac{2\pi \times \frac{1}{2} L I_m^2}{I^2 R T}$$

$$= \frac{\pi L (\sqrt{2} I)^2}{I^2 R T}$$

$$= \frac{\pi L 2 I^2}{I^2 R T} = \frac{\pi L 2 I^2}{I^2 R T} = \frac{2\pi L}{R T}$$

$$\boxed{\text{Quality factor} = \frac{2\pi f_0 L}{R}}$$

Quality factor defined as the reciprocal of P.F.

$$\boxed{Q\text{ factor} = \frac{1}{\cos\phi}}$$

It is reciprocal of selectivity

$$Q\text{ factor or magnification factor} = \frac{\text{voltage across Inductor}}{\text{voltage across resistor}}$$

$$= \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{2\pi f_0 L}{R} = \frac{\omega L}{R} = \text{Quality factor.}$$

$$Q\text{ factor} = \frac{\text{voltage across Capacitor}}{\text{voltage across resistor}} = \frac{I_0 X_C}{I_0 R} = \frac{X_C}{R} = \frac{1}{2\pi f_0 C}$$

$$= \frac{1}{2\pi f_0 C R}$$

$$\boxed{Q\text{ factor} = \frac{1}{\omega_0 C R}}$$

$$Q^2 = \frac{\omega L}{R} \times \frac{1}{\omega C R} \Rightarrow Q^2 = \frac{L}{R^2 C} \Rightarrow Q = \sqrt{\frac{L}{R^2 C}} \Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

CHAPTER-1

TRANSIENTS

⇒ Steady state Response is referred as permanent response because this response doesn't varies with time.

⇒ Transient Response is referred as Temporary response because these response varies with time.

⇒ Transient Response exist due to storing element in the network (Inductor, capacitor).

⇒ Transient response doesn't exist for resistive network.

⇒ Transient Response gives the time constant of the system.

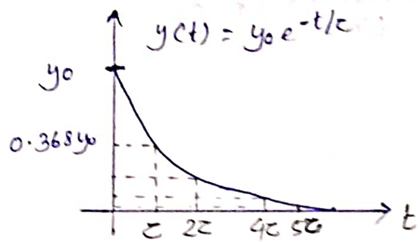
⇒ Transient Response gives the speed of the system.

⇒ Transient Response gives the speed of the system.

Standard discharging equation :-

$$y(t) = y_0 e^{-t/\tau}$$

where, τ = time constant



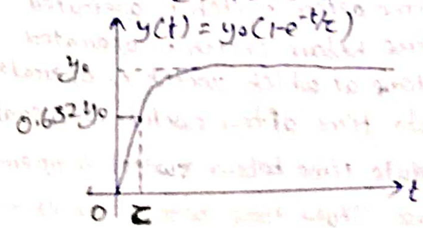
Time Constant (τ) :-

⇒ In case of standard discharging equation, the time at which response will be 36.8% or $\frac{1}{e}$ time of its initial value is referred as "Time Constant".

Standard charging equation :-

$$y(t) = y_0[1 - e^{-t/\tau}]$$

where, τ = Time Constant



Time Constant :-

⇒ In case of standard charging equation, the time at which response will be 63.2% of its final value is referred as

⇒ Generalized equation of both charging and discharging :-

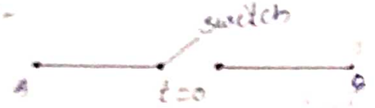
$$\boxed{y(t) = y(\infty) + [y(0) - y(\infty)]e^{-t/\tau}}$$

↳ This eqⁿ is valid only for first order differential eqⁿ.

Ex:- $\boxed{A\frac{dy(t)}{dt} + B\,y(t) = C}$

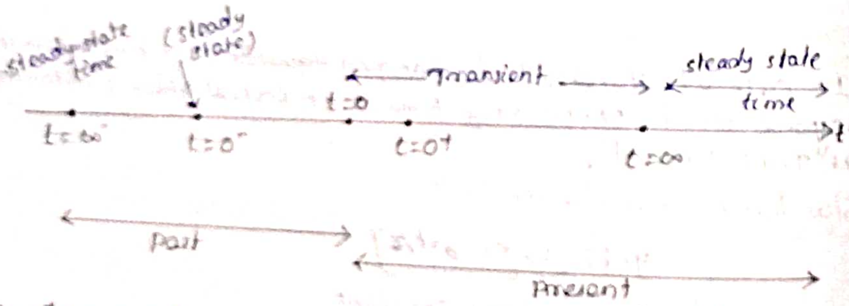
Where, $\tau = A/B$

Switch :-



switching time $[t=0]$

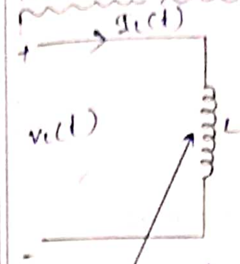
Because all past value are steady state period and steady state not known when the assume I do not (it may be one day before that all deemed means past)



(i) $t=0^+$ represents momentum time after switch is operated.
(ii) $t=0^-$ represents momentum time before switch is operated.
(iii) $t=0$ represents transition time at which switch is operated.
(iv) $t=\infty$ represents steady-state time after switch is operated.
(v) $t=-\infty$ represents steady-state time before switch is operated.
(vi) $t=0^-$ (also) represents steady state time before switch is operated.

At $t=\infty$ → steady state (present)
or $t=0^-$ → steady state (past)

Transient Behaviour of Inductor :-



$V_L(t) = L\frac{d}{dt}I_L(t)$ [Ohm's law]

$I_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$ [Ohm's law]

$I_L(t) = \frac{1}{L}\int_{-\infty}^{0} v_L(t)\,dt + \frac{1}{L}\int_{0}^{t} v_L(t)\,dt$

$I_L(t) = I_L(0^-) + \frac{1}{L}\int_{0}^{t} v_L(t)\,dt$

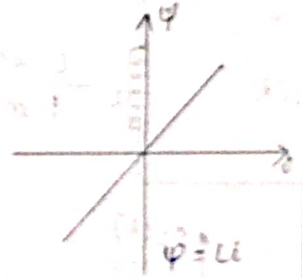
$I_L(0^-)$ = Initial current of Inductor

$E_L(0^-) = \frac{1}{2}L(I_L(0^-))^2$ Joule

$E_L(0^-)$ = Initial Energy of Inductor

Property of Inductor :-

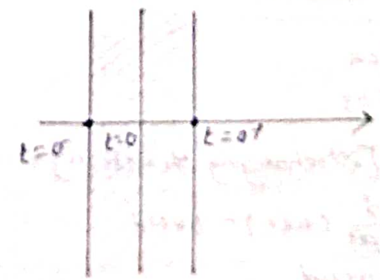
1. It opposes the change of current
$\boxed{I_L(0^+) = I_L(0^-)}$
2. It opposes the change of energy
$\boxed{E_L(0^+) = E_L(0^-)}$
3. It allow sudden change in voltage
$v_L(0^+) \neq v_L(0^-)$
4. It oppose the change in flux



Inductor :-

$I_L(0^-) = I_L(0^+)$
$E_L(0^-) = E_L(0^+)$

In Inductor, from $t=0^-$ to $t=0^+$, current & Energy are same.



→ Steady-state Behaviour of Inductor :-

Case-1

Charging of Inductor :-

(i) $I_L(t)\uparrow$ $I_L(\infty)$ = maximum

(ii) $E_L(t)\uparrow$ $E_L(\infty)$ = maximum

(iii) $V_L(t) = L\frac{dI_L(t)}{dt} = L\frac{d}{dt}$ [Increasing function] = +ve value

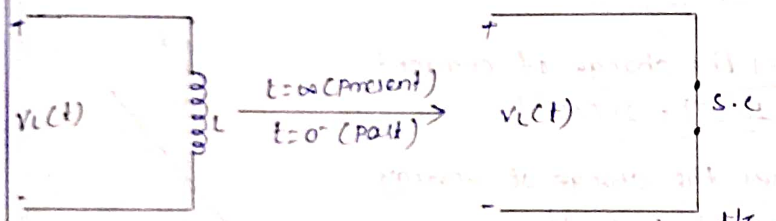
(iv) $V_L(\infty) = L\frac{dI_L(\infty)}{dt} = L\frac{d}{dt}$ [max^m value] = 0 volt

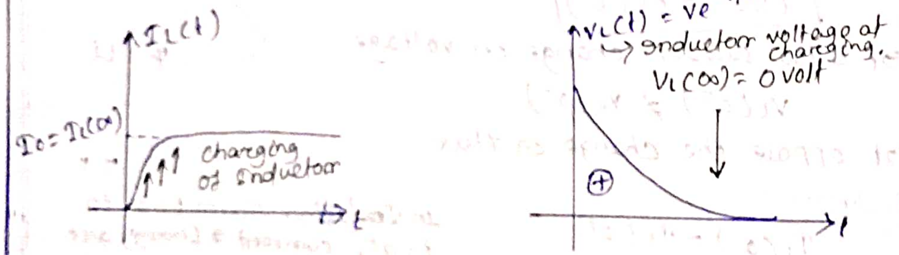
(v) L (Inductor) $\xrightarrow{s.s}$ S.C [short circuit]

$t=\infty$

$t=0$

steady-state condition





Case-2

Discharging of Inductor :-

(i) $I_L(t)\downarrow$ $I_L(\infty)$ = min / 0A

(ii) $E_L(t)\downarrow$ $E_L(\infty)$ = min / 0J

(iii) $V_L(t) = L\frac{dI_L(t)}{dt} = L\frac{d}{dt}$ [Discharging function] = -ve

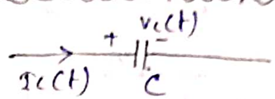
(iv) $V_L(\infty) = L\frac{d}{dt}I_L(\infty) = L\frac{d}{dt}$(min) = 0 volt

(v) $L \xrightarrow{s.s}$ short circuit

$t=\infty$

$t=0^-$

Transient Behaviour of capacitor :-



[Capacitor]

According to ohm's law :- $I_c(t) = C\frac{d}{dt}V_c(t)$

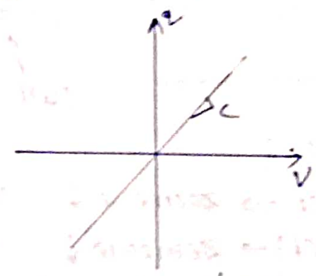
$i(t) = \frac{dq}{dt}$

$q(t) \propto V_c(t)$

$q(t) = CV_c(t)$

$\therefore i(t) = I_c(t)$

$$\boxed{I_c(t) = C\frac{dV_c(t)}{dt}}$$



(capacitor plane)

→ $V_c(t) = \frac{1}{C}\int_{-\infty}^{t} I_c(t)\,dt$

$V_c(t) = \frac{1}{C}\int_{-\infty}^{0} I_c(t)\,dt + \frac{1}{C}\int_0 I_c(t)\,dt$

$$\boxed{V_c(t) = V_c(0^-) + \frac{1}{C}\int_0^t I_c(t)\,dt}$$

$V_c(0^-)$ = Initial voltage of capacitor

→ $\boxed{E_c(0^-) = \frac{1}{2}CV_c(0^-)^2 \text{ Joule}}$ [Initial energy of capacitor]

Property of capacitor :-

→ It opposes of change of voltage i.e $V_c(0^+) = V_c(0^-)$

→ It opposes of change of charge i.e $q(0^+) = q(0^-)$

→ It opposes of change of energy i.e $E_c(0^+) = E_c(0^-)$

$E_c = f(V_c)$

→ It allows sudden change in current i.e $I_c(0^+) \neq I_c(0^-)$

Steady-state Behaviour of capacitor :-

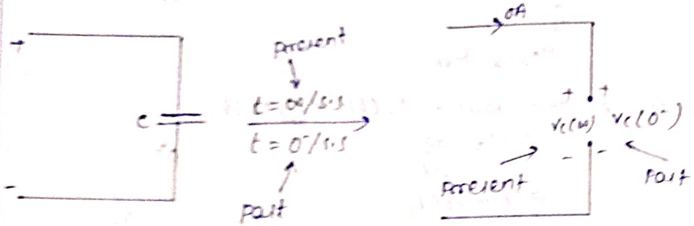
Case-1 Charging of capacitor :-

Condition (i) $V_c(t) \rightarrow$ Increase ↑ $V_c(\infty)$ = maximum

(ii) $E_c(t) \rightarrow$ increase ↑ $E_c(\infty)$ = maximum

(iii) $I_c(t) = C\frac{dV_c(t)}{dt}$ = +ve

(iv) $I_C(\infty) = C\frac{dv_c(\infty)}{dt} = C\frac{d}{dt}(max^m) = 0$ [open ckt]



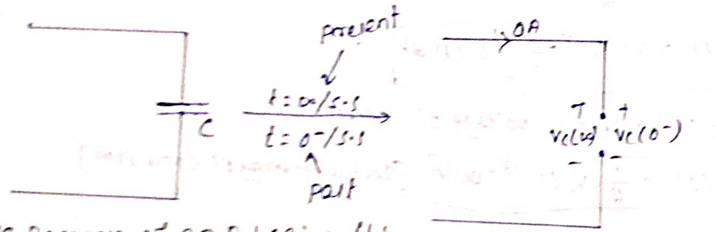
Case-2

(i) $v_c(t) \rightarrow$ decrease ↓ $v_c(\infty)$ = minimum/0.V

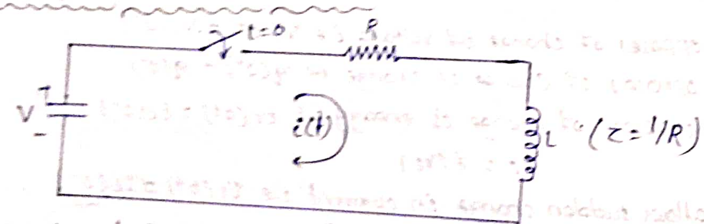
(ii) $E_c(t) \rightarrow$ decrease ↓ $E_c(\infty)$ = min^m/0J

(iii) $I_c(t) = C\frac{dv_c(t)}{dt} = -ve$

(iv) $I_c(\infty) = C\frac{dv_c(\infty)}{dt} = C\frac{d}{dt}$(minimum) = 0A [open circuit]



DC Response of an R-L circuit :-



Generalized Transient eq^n :-

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-Rt/L}$$

$$i(0^+) = i(0^-) = 0A$$

$$i(\infty) = \frac{V}{R}$$

$$i(t) = \frac{V}{R} + \left[0 - \frac{V}{R}\right]e^{-Rt/L}$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-Rt/L}\right]}$$ → charging eq^n

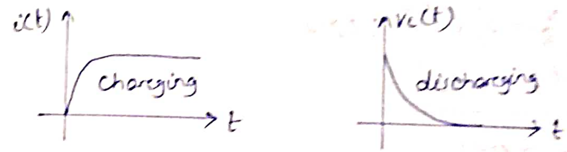
Inductor voltage :- (V_L)

$$V_L = L\frac{di(t)}{dt}$$

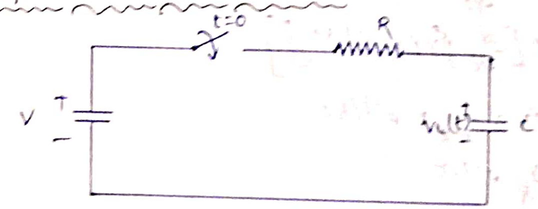
$$V_L = L\frac{d}{dt}\left(\frac{V}{R} - \frac{V}{R}e^{-Rt/L}\right)$$

$$V_L = \frac{LV}{R}\left(0 + \frac{R}{L}e^{-Rt/L}\right)$$

$$\boxed{V_L = Ve^{-Rt/L}}$$ → discharging eq^n



DC Response of an R-C circuit :-



Generalised Transient Eq^n :-

$$V_c(t) = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} \; ; t \geq 0$$

→ $v_c(0^+) = v_c(0^-) = 0V$

→ $\tau = R_{TH}C$ sec.

→ $v_c(\infty) = V$

→ $I_c(\infty) = 0A$

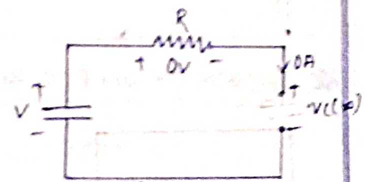
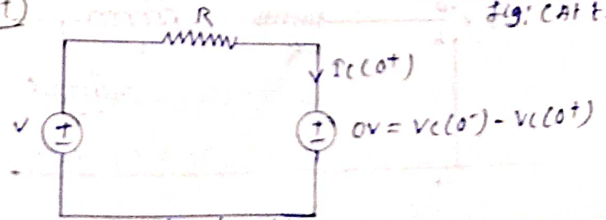


fig: (At $t=\infty$)

$I_c(0^+)$



(At $t = 0^+$)

KVL

$-V + I_C(0^+)R + 0 = 0$

$\Rightarrow I_C(0^+) = \frac{V}{R}$ Amp

→ $v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]e^{-t/\tau}$; $t \geq 0$

$v_C(t) = V + [0 - V]e^{-t/\tau}$

$\boxed{v_C(t) = V[1 - e^{-t/RC}]}$

→ $I_C(t) = I_C(\infty) + [I_C(0^+) - I_C(\infty)]e^{-t/\tau}$; $t \geq 0$

$I_C(t) = 0 + [\frac{V}{R} - 0]e^{-t/\tau}$; $t \geq 0$

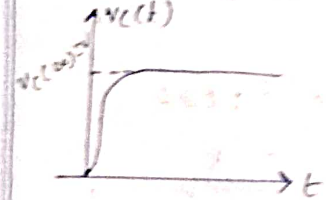
$\boxed{I_C(t) = \frac{V}{R}e^{-t/\tau} \; ; t \geq 0}$

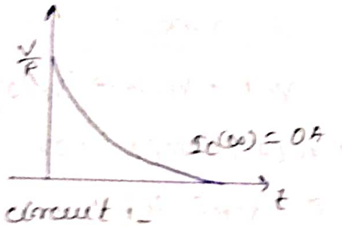
or $I_C(t) = C\frac{dv_C(t)}{dt}$

$= C[\frac{d}{dt}\{V(1 - e^{-t/RC})\}]$

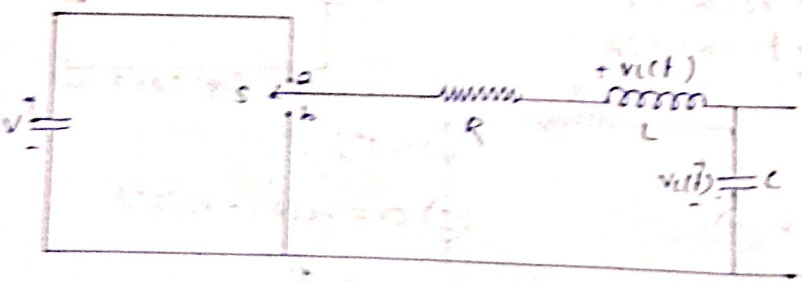
$= \cancel{C}V \times \frac{1}{R\cancel{C}} \times e^{-t/RC}$

$I_C(t) = \frac{V}{R}e^{-t/\tau}$

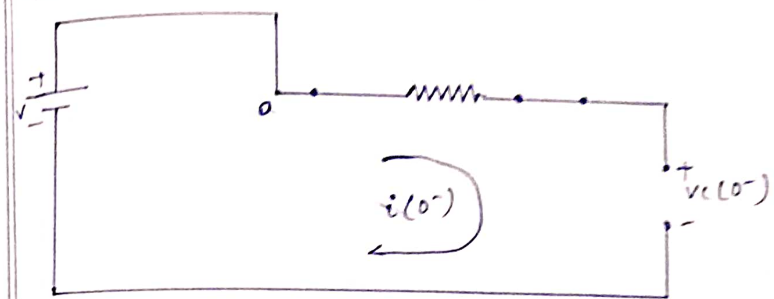




DC Response of an RLC series circuit :-



at $t = 0^-$/ s.s S → "a" (switch)



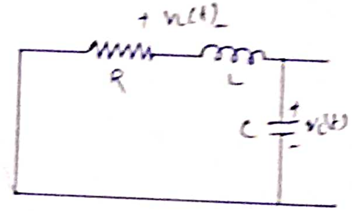
$I_L(0^-) = i(0^-) = 0A$

Applied KVL for $v_C(0^+)$

KVL

$\Rightarrow -V + 0 \times R + v_C(0^-) = 0$

$\Rightarrow v_C(0^-) = V$ volt



at $t = 0$ S → a to b

$i(0^+) = I_L(0^+) = I_C(0^+) = 0A$

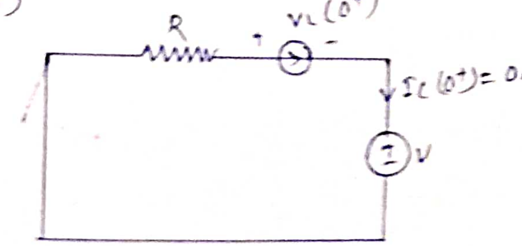
$\Rightarrow I_C(0^+) = I_C(0^-)$ $[I_C(t) = I_L(t)]$

$= 0A$

$\Rightarrow v_C(0^+) = v_C(0^-) = V$ volt

At $t = 0^+$

$v_L(0^+)$



KVL

→ $0 \times R + v_L(0^+) + V = 0$

$\Rightarrow v_L(0^+) = -V$ volt.